UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2007/2008

TITLE OF PAPER	:	CLASSICAL MECHANICS
COURSE NUMBER	:	P320
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS		ANSWER ANY FOUR QUESTIONS – EACH QUESTION CARRIES 25 MARKS – MARKS FOR EACH SECTION ARE SHOWN IN THE RIGHT-HAND COLUMN

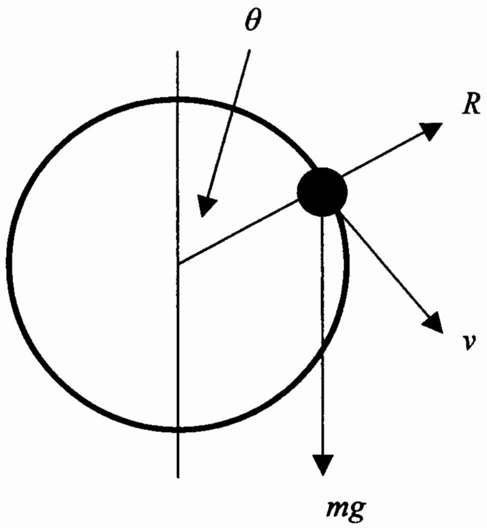
THIS PAPER HAS 8 PAGES INCLUDING THIS PAGE

A table of the values of the physical constants is provided

Question 1

A particle is in equilibrium. What does this statement mean? [2]

A bead can slide on a smooth circular wire of radius a, which is fixed in a vertical plane. The bead is displaced slightly from the highest point of the wire. Find the speed of the bead as a function of θ. [10]



The bead experiences two forces; R, the reaction force, and mg, and the force due to gravity. What is the work done by R on the bead? [5]

[hint: the bead is always moving tangentially to the wire]

What is the Lagrangian for the bead? [5]

The bead passes through the lowest point with a finite velocity. What height will the bead reach on the left-hand side of the ring? [3]

Question 2

Consider 2 particles at positions P_1 and P_2 with masses m_1 and m_2 respectively. Their centre of mass G is the point situated along the line connecting the two masses, between P_1P_2 , such that

$$P_1G = \frac{m_2}{m_1 + m_2} P_1P_2, \text{ and similarly } GP_2 = \frac{m_1}{m_1 + m_2} P_1P_2 .$$

For many particles, we can generalize the above expressions, and obtain

$$\mathbf{G} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + ...}{m_1 + m_2 + ...}.$$

Show that the position of **G** is independent of the origin of the co-ordinate system. [5]

[hint: shift the origin, so that $\mathbf{r}_i$ become $\mathbf{r}'_i = \mathbf{r}_i + \mathbf{\Delta}$, and work through the algebra]

Using a two-particle model for the earth-moon system, show that the earth and moon revolve about a point that is approximately 2,900 miles (4,640 km) from the centre of the earth. The mass of the earth is 81 times that of the moon and the distance between them is 240,000 miles or 384,000 km. List the assumptions you have made in obtaining this result. [10]

[just for interest: the mean radius of the earth is 4,000 miles or 6,370 km.]

In the centre of mass frame of the earth-moon system, both bodies move in ellipses about the centre of mass, with semi-major axes of

$$a_1 = \frac{m_2}{M} a \quad \text{and} \quad a_2 = \frac{m_1}{M} a$$

where $M = m(\text{earth}) + m(\text{moon})$, $a = 3.8 \times 10^5$km., $a_1 =$ the distance of the moon from the centre of the centre-of-mass of the moon-earth system, $a_2 =$ the distance of the earth from the centre of the centre-of-mass of the moon-earth system, $m_1 =$ the mass of the moon, $m_2 =$ the mass of the earth.

This leads to a small oscillation in the apparent direction of the sun (as viewed from the earth). The distance of the earth-moon system from the sun is about 1.5×10^8km. What is the angular amplitude of the oscillation as viewed from the earth?
[10]

Question 3

Estimate the mass of the earth, given that the acceleration induced by the gravity of the earth at its surface is 9.81 m/sec^2. List any assumptions you make. [5]

A rocket is fired from the surface of the earth. What is the lowest speed that the rocket must have so that it just escapes the earth's gravitational field. Neglect the rotation of the earth and air resistance. [10]

If the projectile is launched with a velocity less than the escape velocity, it will reach a maximum height and then fall back to earth. Derive an expression for the maximum distance between the earth and the rocket. [10]

The gravitational constant, G, is 6.672 x 10^{-11} N m^2 kg^{-2}
The mean radius of the earth is 6.4 x 10^3 km

Question 4

Use Lagrangian mechanics to determine the period of a simple harmonic oscillator that undergoes small oscillations about its position of equilibrium. [25]

Question 5

Consider the rotation of a rigid body about a fixed axis chosen to be the z-axis. The origin of the co-ordinate system lies on this axis such that the z co-ordinate of the centre of mass of the body is zero. Using cylindrical polar co-ordinates, $(\rho\ \varphi\ z)$, ρ and z of every part of the body are fixed, only φ changes with time.

Show that $\mathbf{v} = \omega \times \mathbf{r}$, where $\mathbf{v}$ is the velocity of a small part of the rigid body, and $\mathbf{r}$ is its distance from the origin of the co-ordinate system. What is the relationship between ω and φ? [10]

If the angular momentum $\mathbf{J}_z = \mathrm{I}\boldsymbol{\omega}$, where I is the moment of inertia, derive an expression for the rotational kinetic energy of the body in terms of I and ω. [5]

For a body rotating about the z-axis, there may be components of $\mathbf{J}$ perpendicular to the z direction, and one can write

$J_x = I_{xz}\omega \qquad J_y = I_{yz}\omega \qquad J_z = I_{zz}\omega.$

By considering the rate of change of the three Cartesian co-ordinates, because of the rotation of the body, obtain expressions for I_{xz}, I_{yz} and I_{zz}. [10]

Question 6

A tennis ball, mass *m* and diameter 0.1*m*, rolls down an inclined surface. Its centre of gravity starts at height *h* above the laboratory bench. What is the change in the height of the centre of gravity of the tennis ball? [5]

What is the velocity of the tennis ball at the moment it first touches the bench? [15]

Carefully list all the assumptions you have made in your calculation. [5]

Treat the tennis ball as a thin spherical shell which has a moment of inertia $I = \frac{2}{3}ma^2$, where a is the radius of the tennis ball.

PHYSICAL CONSTANTS AND UNITS

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.673×10^{-11} N m^2 kg^{-2}
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
(Note: 1 mole = 1 *gram* molecular-weight)		
Ice point	T_{ice}	273.15 K
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} = 0.862×10^{-4} eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞ $R_\infty hc$	1.097×10^{7} m^{-1} 13.606 eV
Planck constant	h	6.626×10^{-34} J s = 4.136×10^{-15} eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s = 6.582×10^{-16} eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3} = 1/137.0
Compton wavelength of electron	$\lambda_c = h/mc$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
torr (mm Hg, 0°C)	torr	133.32 Pa (N m^{-2})
barn	b	10^{-28} m^2