UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2007/2008

TITLE OF PAPER	**:**	**ELECTROMAGNETIC THEORY**
COURSE NUMBER	**:**	**P331**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS		**ANSWER ANY FOUR QUESTIONS - EACH QUESTION CARRIES 25 MARKS – MARKS FOR EACH SECTION OF EACH QUESTION ARE SHOWN IN THE RIGHT-HAND COLUMN**

THIS PAPER HAS 7 PAGES INCLUDING THIS PAGE

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A table of the values of physical constants is provided in addition to a sheet of equations relevant to the course

Question 1

Consider two parallel, charged, isolated plates in a vacuum. Explain what happens when a dielectric (insulator) is inserted between the plates. [14]

[Explain why the magnitude of the electric field between the plates is reduced, even though the charge density on the plates is unaffected by the presence of the dielectric].

Give two equivalent definitions of the electric polarization, **P**? [4]

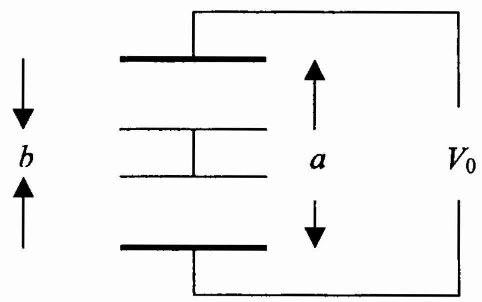
Explain the meaning of the phrase "any discontinuity in **P** is equivalent to a bound charge density". [7]

Question 2

The figure shows two capacitors arranged in series; the rigid central section of length b can be moved vertically, either towards the top plate of the outer capacitor or towards the bottom plate. The separation of the outer two plates is a. The areas of the four plates are each equal to A. Show that the capacitance of the series combination is independent of the position of the central section and is given by $C = \frac{\varepsilon_0 A}{a-b}$. [10]

If the voltage difference between the outside plates, heavy lines in the figure, is kept constant at V_0, what is the *change* in energy stored in the capacitor if the central section is removed? [15]

[the capacitance of two capacitors in series is $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2}$ and the energy stored in a capacitor = $\frac{1}{2}CV^2$]



Question 3

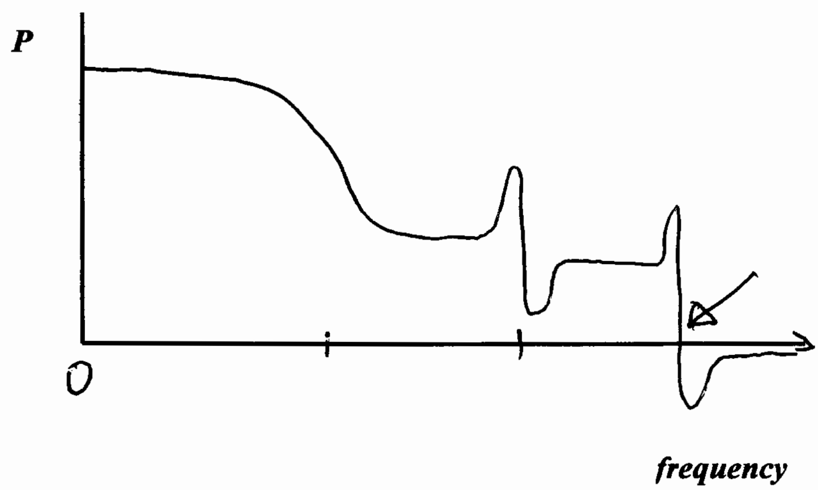
Equation 2 on the sheet of equations provided implies that magnetic monopoles do not exist, i.e. that $\nabla.\mathbf{B} = 0$. Write down an expression for the relationship between the magnetic induction, **B**, and the vector potential **A**. Show that the expression involving **A**, does indeed give $\nabla.\mathbf{B} = 0$. [7]
[Work through the algebra involving the two vector operators that act on **A**].

By considering conservation of charge, show that Ampère's Law can only apply to steady currents (currents that do not vary with time). [6]
Maxwell modified Ampère's law by making a hypothesis concerning a vector whose divergence he showed must be zero. What was that vector, and what did Maxwell assume? [12]

Question 4

Using equation 10 on the sheet, show that for a non-magnetic insulator, one would expect $n^2 = \varepsilon_r$, where n is the refractive index and ε_r, the relative permittivity. [10]

The frequency dependence of the polarization, ***P***, for many materials (such as water), has the following form



Explain the shape of this curve, giving the details of the various contributions made to $\boldsymbol{P}$ as frequency changes. What are the frequencies at which the relatively abrupt changes take place? [10]

For water, the refractive index is about 1.3 while experimentally the relative permittivity is about 81. Explain why the relationship given in the first paragraph of this question appears not to be true. [5]

Question 5

From manipulation of suitable equations from the sheet of equations, derive an expression for the phase and group velocity of light in free space. [15]

Comment on any attenuation of the wave. [5]

From expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$ obtain the magnitude of the impedance of free space. [5]

Question 6

We used a simple model to obtain a frequency-dependent expression for the electrical conductivity of a conductor. We assumed that the positive ions vibrate about fixed positions, and that the current is carried by mobile electrons – or conduction electrons – and that typically there are one or two conduction electrons per positive ion. The mobile electrons suffer collisions on average every τ seconds. Show that this can lead to an expression for the conductivity, σ,

$$\sigma = \frac{ne^2\tau}{m[1 + j\omega\tau]}$$

where n is the number density of mobile charge carriers, m is the mass of the electron, and the other symbols have their usual meaning. Explain carefully any assumptions that you made in deriving the expression for the conductivity. [15]

Given that the d.c. *resistivity* of copper at room temperature is about 1.7 x 10^{-8} Ω.m, make an estimate for τ at this temperature. [5]

What is the value for the high-frequency expression for σ? [2]

In this limit, are collisions experienced by the conduction electrons important? [3]

[n for copper is approximately 2 x 10^{28} m^{-3}]

PHM31E ELECTROMAGNETISM & RELATIVITY

Important equations required to describe the propagation of electromagnetic radiation in matter.

1 $\nabla.(\varepsilon\mathbf{E}) = \rho_f$

2 $\nabla.\mathbf{H} = 0$

3 $\nabla\mathrm{x}\mathbf{E} + \mu\dot{\mathbf{H}} = 0$

4 $\nabla\mathrm{x}\mathbf{H} - \varepsilon\dot{\mathbf{E}} = \mathbf{J}_f = \sigma\mathbf{E}$

$\nabla\mathrm{x}\nabla\mathrm{x}\mathbf{E} = -\nabla^2\mathbf{E} + \nabla(\nabla.\mathbf{E})$ cartesian co-ordinates

5 $\nabla^2\mathbf{E} - \mu\varepsilon\ddot{\mathbf{E}} - \mu\sigma\dot{\mathbf{E}} = 0$

6 $\nabla^2\mathbf{H} - \mu\varepsilon\ddot{\mathbf{H}} - \mu\sigma\dot{\mathbf{H}} = 0$

7 $\mathbf{E} = \mathbf{E}_0 \exp j(\omega t - kz)\,\mathbf{i}$

8 $\mathbf{H} = (k/\omega\mu)\mathbf{E}_0 \exp j(\omega t - kz)\,\mathbf{j}$

9 $-k^2 + \omega^2\varepsilon\mu - j\omega\sigma\mu = 0$

10 $k^2 = \varepsilon_r\mu_r k_0^2[1 - j\sigma/\omega\varepsilon]$

$k_0 = \omega/c$

$c = (\mu_0\varepsilon_0)^{-1/2}$

$\mathbf{S} = \mathbf{E}\mathrm{x}\mathbf{H}$ Poynting's vector

$E/H = \omega\mu/k$ wave impedance

PHYSICAL CONSTANTS AND UNITS

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.673×10^{-11} N m^2 kg^{-2}
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
	(Note: 1 mole = 1 *gram* molecular-weight)	
Ice point	T_{ice}	273.15 K
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} = 0.862×10^{-4} eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s = 4.136×10^{-15} eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s = 6.582×10^{-16} eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3} = 1/137.0
Compton wavelength of electron	$\lambda_c = h/mc$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
torr (mm Hg, 0°C)	torr	133.32 Pa (N m^{-2})
barn	b	10^{-28} m^2

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