UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2008.

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) Sate two properties of a ***primitive*** unit cell. (2 marks)

(b) Draw ***one*** figure showing a conventional unit cell and also its primitive cell of an fcc lattice

(4 marks)

(c) One side of a conventional unit cell of an f.c.c. lattice is 3 Å . What is the volume of its primitive unit cell?

(2 marks)

(d) Calculate the separation between two (123) planes of an orthorhombic lattice with cell lengths, a = 0.82 nm, b = 0.94nm and c = 0.75nmin the x,y, z directions.

(3 marks)

(e) Compute the packing fraction of a b.c.c lattice. (4 marks)

(f) Find the indices of the (100) planes of an f.c.c lattice as referred to its primitive axes.

(4 marks)

(g) Show that the reciprocal lattice of a b.c.c is f.c.c. (6 marks)

Given:

Primitive translation vectors of fcc are:

$$a_1 = \frac{1}{2}a(\hat{x} + \hat{y}), b_1 = \frac{1}{2}a(\hat{y} + \hat{z}), c_1 = \frac{1}{2}a(\hat{z} + \hat{x})$$

Primitive translation vectors of bcc are:

$$a_1 = \frac{1}{2}a(\hat{x} + \hat{y} - \hat{z}), b_1 = \frac{1}{2}a(-\hat{x} + \hat{y} + \hat{z}), c_1 = \frac{1}{2}a(\hat{x} - \hat{y} + \hat{z})$$

Question Two.

(a) Obtain an expression for the dispersion relation for a one-dimensional monatomic linear lattice of lattice constant "a", atomic mass "M" and force constant "C"

(15 marks)

(b) (i) Draw a sketch showing how the phonon frequency varies with wave vector in the first Brillouin zone. (4 marks)

(ii) What are the values of the frequency for k =0 and k= π / a ?

(2 marks)

(c) Show that when the phonon wavelength is large compared to the interatomic spacing the phase velocity $\frac{\omega}{K} = a\sqrt{\frac{c}{M}}$ where "c" is the interatomic force constant for nearest neighbours. (4 marks)

Question Three.

(a) State the assumptions Drude made in his ***free electron theory*** of metals. (3 marks)

(b) Define the terms ***mean free path*** and ***mobility*** of an electron. (2+2 marks)

(c) Show that according to Drude theory the d.c electrical conductivity of a metal can be expressed as:

$$\sigma = \frac{ne^2\tau}{m},$$ where symbols have their usual meanings. (10 marks)

(d) (i) State ***Wiedemann - Franz*** law. (3marks)

(ii) Write down the expression for ***Lorenz number*** and calculate its value. (2 + 3 marks)

Question Four.

(a) (i) Define the terms: "density of states" and "Fermi energy". (4 marks)

(ii) Derive an expression for the density of states of a system of electrons given that the Fermi energy:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$ (6 marks)

(iii) Calculate the density of energy states at 2.05 eV energy for a system of electrons in a volume of 1 cm^3. (8 marks)

(b) (i) Show that the electronic contribution to heat capacity of a metal is proportional to absolute temperature. (4 marks)

(ii) Discuss the heat capacity of metals, explaining difference if any between the above theory and experimental values. (3 marks)

Question Five.

(a) Giving silicon as an example explain how electrical conductivity of a semiconductor can be increased by doping.

(6 marks)

(b) With the help of appropriate diagram, derive an expression for the effective density of states in the conduction band of a semiconductor. Asume: $(\epsilon-\epsilon_F) \gg kT$.

(10 marks)

Given:

Fermi -Dirac distribution function: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/kT}+1}$

Density of states for a system of fermions:

$$D(\varepsilon)d\varepsilon = \frac{4\pi}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

(c) A doped semiconductor has electron and hole concentrations $2\text{x}10^{13}$ cm^{-3} and $1.41\ \text{x}10^{13}$ cm^{-3} respectively. Calculate the electrical conductivity of the sample. (5 marks)

[Take: $\mu_n = 4200\ cm^2\ V^{-1}s^{-1}$. $\mu_p = 2000\ cm^2\ V^{-1}s^{-1}$]

(d) Discuss briefly the process of photoconductivity in semiconductors. (4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-\alpha x^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-\alpha x^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-\alpha x^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-\alpha x^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-\alpha x^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ l\ mol^{-1}$