UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2008/09

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Prove the law of sines using the cross product. Hint : Consider the area of a triangle formed by vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$. **(6 marks)**

(b) For a particle moving in a circle of radius R at constant angular velocity ω, along the xy plane, show that the velocity $\vec{\upsilon}$ is perpendicular to the position $\vec{r}$. **(5 marks)**

(c) Write down the acceleration in plane polar coordinates and state the meaning of each term. **(6 marks)**

(d) A projectile if fired up an incline (incline angle ϕ) with an initial speed υ_0 at an angle θ with respect to the horizontal ($\theta > \phi$), as shown in Figure 1. Show that the height h is given by

$$h = \upsilon_0 R \tan\theta - \frac{gR^2}{2\upsilon_0^2 \cos^2\theta}.$$ **(8 marks)**

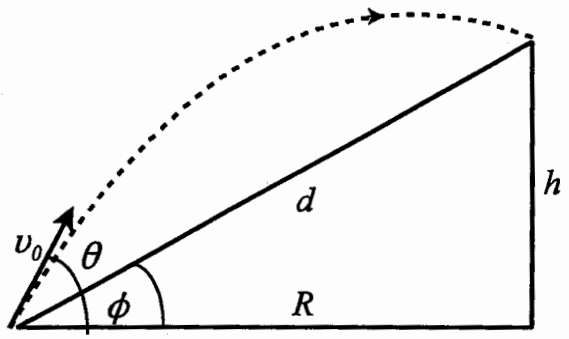

Figure 1.

QUESTION 2

(a) Consider a body of mass m on which the net force $f = -km\upsilon^2$ is a resistive force proportional to the square of its speed υ. Suppose it is at the origin $x = 0$ at $t = 0$ and is given an initial velocity υ_0 along the positive x-axis.

(i) What is its velocity $\upsilon(t)$ at a later time t? **(4 marks)**
(ii) What is its position at a later time t, $x(t)$? **(4 marks)**
(iii) What is the body's velocity as a function of position x? **(5 marks)**

(b) A block moves around an inclined plane of radius R and banking angle θ.. See Figure 2. The coefficient of static friction up or down the inclined plane between the block and the incline is μ. Find an expression for the minimum and maximum velocities for the block to go around the curve without slipping downward or upward the inclined plane, respectively. **(12 marks)**

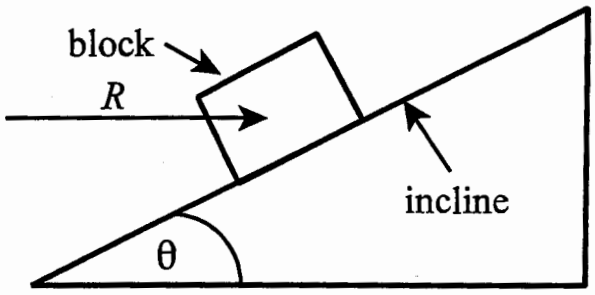

Figure 2.

QUESTION 3

(a) A thin uniform rod of length L and mass M is made into a curve of radius r subtending an angle of 90° as shown in Figure 3.

(i) Write r in terms of L. **(2 marks)**

(ii) Write the mass dm of an infinitesimal element of the rod subtended by the angle $d\theta$ in terms of M, r, L and $d\theta$.. **(3 marks)**

(iii) Find the centre of mass of the curved rod and make a diagram to illustrate where it is located. **(10 marks)**

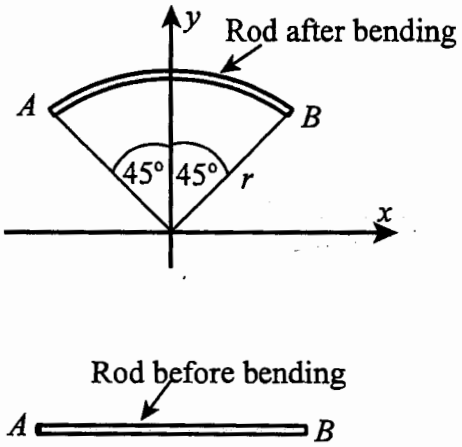

Figure 3.

(b) A space craft of initial mass M_0 initially at rest in free space moves for a time t. During its motion it ejects some exhaust gases with a speed v_e at a rate k kg/s. Use mass transport to get the differential equation that can be used to determine the velocity of the rocket at a later time t. Do not find the velocity. **(10 marks)**

QUESTION 4

(a) A simple pendulum has a mass m suspended on a string of length l. The mass m is swung over an angle θ_0 with the vertical and let go. Use calculus to determine an expression for the work done by the gravitational force gravity mg after the string has swung from θ_0 to θ_f in terms of m, g, l, θ_0 and θ_f. **(7 marks)**

(b) A body on a planet's surface of radius R is projected upward with an initial velocity υ_0. The gravitational force varies with altitude the normal way. Let M_p and m be the masses of the earth and projectile, respectively. Use the work-energy theorem to determine a general expression for the highest point reached by the projectile. **(9 marks)**

(c) A body of mass m moves along the x-axis under the force

$$F = -Ax^2 + Bx,$$

where A, and B are positive constants.

(i) What is the potential energy function? **(3 mars)**

(ii) Determine the equilibrium point(s) for the body. **(2 marks)**

(iii) Determine the stability of the equilibrium point(s). **(2 marks)**

(iv) What is the angular frequency of small oscillations about the stable equilibrium point? **(2 marks)**

QUESTION 5

(a) A thin right angle triangular uniform plate of width w, base L and height h and mass M is to be rotated about an axis through one of its vertices and perpendicular to the side of length L and parallel to one of its sides as shown in Figure 4.

(i) Find the infinitesimal volume element dm in terms of M, x, L and dx. **(4 marks)**

(ii) Find its moment of inertia in terms of M and L. **(4 marks)**

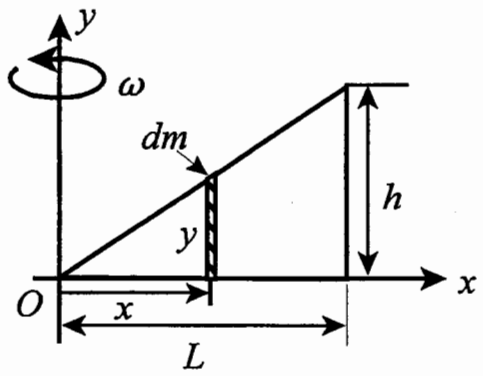

Figure 4.

(b) (i) Derive with the aid of diagrams an equation of motion for a physical pendulum and show that it oscillates. **(6 marks)**

(ii) Determine the angular frequency of the oscillation in terms of the radius of gyration k, the distance of the pivot from the centre of mass l, and the gravitational acceleration g. **(4 marks)**

(iii) Use the equation obtained in (ii) to determine the angular frequency for a simple pendulum. **(2 marks)**

(c) Show that the torque $\vec{\tau}$ and angular momentum $\vec{L}$ are related by the following equation:

$$\vec{\tau} = \frac{d\vec{L}}{dt}.$$ **(5 marks)**