

**UNIVERSITY OF SWAZILAND**

**FACULTY OF SCIENCE**

**DEPARTMENT OF PHYSICS**

**MAIN EXAMINATION 2008/09**

**TITLE OF PAPER:** MODERN PHYSICS & WAVE OPTICS

**COURSE NUMBER:** P231

**TIME ALLOWED:** THREE HOURS

**INSTRUCTIONS:** ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS FORMULAE AND CONSTANTS THAT MAY BE USEFUL IN SOME PROBLEMS

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### QUESTION 1.

(a) Light from a helium-neon laser at the wavelength  $\lambda = 632.8 \text{ nm}$  is incident on a single slit of width  $a = 0.200 \text{ mm}$ . The viewing screen is placed a distance  $L = 2.5 \text{ m}$  from the slit.

(i) Make a diagram that illustrates the intensity distribution for the diffraction pattern of a single slit of width  $a$  on a screen a distance  $L$  from the slit. (3 marks)

(ii) Find the position of the first dark fringe along the screen. (6 marks)

(iii) What is the width of the central maximum? Also compare this width to the size of the slit. (2 marks)

(iv) Suppose the light from the same laser is directed to a slit of width  $a = 10 \text{ mm}$  at the same distance from the viewing screen. Determine the position of the first dark and second dark fringes in this case and explain how the image looks. (4 marks)

(b) An oil film of thickness  $t = 600 \text{ nm}$  with a refractive index  $n_{\text{oil}} = 1.60$  floats on water of refractive index  $n_{\text{water}} = 1.33$  and is illuminated by a continuous spectrum from the sun at midday. Some colours appear to be very strong compared to others when viewing the film from directly above. Determine which visible wavelengths are enhanced by the film. Above the film is air of refractive index  $n_{\text{air}} = 1.00$ . (10 marks)

## QUESTION 2

(a) A beam of unpolarised light travels along the  $y$ -axis towards a polarising filter, which is in the  $x$ - $z$  plane with its axis aligned with the  $z$ -direction. The incident beam has intensity  $I_0$ .

- (i) What is the intensity of the emergent beam from the filter. **(7 marks)**
- (ii) Describe the nature of the electric field of the emergent beam. **(2 marks)**
- (iii) If a second polariser is placed after the first at an angle  $\theta = 30^\circ$  with the first. Find the intensity of the emergent beam and describe the nature of the electric field. **(3 marks)**

(b) Fully discuss with the aid of diagrams and by listing the important points the process of polarization by double refraction (birefringence). **(13 marks)**

### QUESTION 3

- (a) The classical model of blackbody radiation given by Rayleigh-Jeans law has two major flaws. Identify them and explain how Planck dealt with them. **(6 marks)**
- (b) Two light sources are used in the photoelectric experiment to determine the work function of a particular metal surface. When green light from a mercury lamp ( $\lambda = 546.1 \text{ nm}$ ) is used a stopping potential of  $0.376 \text{ V}$  reduces the photocurrent to zero.
- (i) Calculate the work function for the metal in electron volts. **(3 marks)**
  - (ii) What stopping potential would be observed when using the yellow light from a helium discharge lamp ( $\lambda = 587.5 \text{ nm}$ )? **(3 marks)**
- (c) X-rays having an energy of  $300 \text{ keV}$  undergo Compton scattering from a target. The scattered rays are detected at an angle of  $37.0^\circ$  relative to the incident rays.
- (i) Find the Compton shift at this angle. **(3 marks)**
  - (ii) What is the wavelength of the scattered X-rays? **(3 marks)**
  - (iii) What is the energy of the scattered X-rays? **(2 marks)**
  - (iv) What is the energy of the recoiling electron? **(2 marks)**
- (d) A vehicle of mass  $1000 \text{ kg}$  moves at a speed of  $250 \text{ kilometres per hour}$ . Find its de Broglie wavelength of the car and state whether it would be possible to measure such a wavelength. **(3 marks)**

#### QUESTION 4

(a) Show that the speed of the electron in the  $n^{\text{th}}$  Bohr orbit in hydrogen is given by

$$v_n = \frac{k_e e^2}{n\hbar},$$

where the symbols have their usual meaning. Justify all your steps.

**(8 marks)**

(b) Consider a hydrogen atom in a highly excited state of  $n = 273$  that makes a transition to the level  $n = 272$ .

(i) Use Balmer's and Rydberg equation to find the wavelength of the emitted radiation.

**(3 marks)**

(ii) Treat the electron in the  $n = 273$  level classically and determine the wavelength of the radiation it emits.

**(5 marks)**

(iii) State Bohr's correspondence principle and use it to explain the results obtained in (i) and (ii).

**(4 marks)**

(c) make a diagram that illustrates how lasing is achieved in a helium-neon (He-Ne) laser.

**(5 marks)**

## QUESTION 5

(a) List the five pathways by which natural radioactivity occurs. Also write down the complete relevant equations making  $X$  the symbol for the original element and  $Y$  the daughter element. Number these pathways (i) to (v). **(10 marks)**

(b) Between the reactions shown below in (i) and (ii), determine by calculating the disintegration energy  $Q$  which reaction can occur spontaneously and which one cannot. Also state which reaction is endothermic and which is exothermic, while giving the meaning of each of these terms.



(c) A wooden spoon is found in a cave and contains 21.0 mg of pure carbon. When the sample is placed in a well calibrated beta counter it is found to have an activity of 951 decays a week. Assuming that the ratio of carbon 14 to carbon 12 has not changed appreciably since the wood was cut. Further assume that the spoon was carved from freshly cut wood.

- (i) What is the total number of carbon nuclei in the sample? **(1 mark)**
- (ii) What was the original number of carbon 14 nuclei in the sample? **(3 marks)**
- (iii) What is the decay constant for the sample? **(1 mark)**
- (iv) What was the initial activity of the sample? **(1 mark)**
- (v) How long ago was the spoon carved in years? **(3 marks)**

## SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number  $A = 6.02 \times 10^{23}$  particles per mole

$$\sigma = 5.6696 \times 10^{-8} \text{ W}/(\text{m}^2\text{K}^2)^3$$

Coulomb constant  $k_e = 8.987551788 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$

Boltzmann's constant,  $k_B = 1.3801 \times 10^{-23} \text{ J/K}$

Bohr radius  $a_0 = 5.291772 \times 10^{-11} \text{ m}$

Bohr magneton,  $\mu_B = 9.27 \times 10^{-24} \text{ J/T}$

Radii of orbit for the hydrogen atom  $r_n = n^2 a_0$

Planck's constant,  $h = 6.626075 \times 10^{-34} \text{ Js}$

$$\hbar = 1.054572 \times 10^{-34} \text{ Js}$$

$$hc = 1.986447 \times 10^{-25} \text{ Jm}$$

$$2\pi\hbar c^2 = 3.741859 \times 10^{-15} \text{ Jm}^2\text{s}^{-1}$$

Rydberg constant  $R_H = 1.097373 \times 10^7 \text{ m}^{-1}$ .

Speed of light in vacuum,  $c = 2.99792458 \times 10^8 \text{ m/s}$

mass of an electron,  $m_e = 9.1093897 \times 10^{-31} \text{ kg}$

mass of a proton,  $m_p = 1.672623 \times 10^{-27} \text{ kg}$

mass of a neutron,  $m_n = 1.6749286 \times 10^{-27} \text{ kg}$

Coulomb constant,  $k_e = 8.987551787 \times 10^9 \text{ Nm}^2/\text{C}^2$

electron charge,  $e = 1.60217733 \times 10^{-19} \text{ C}$

1 atomic mass unit = 1 amu = 1 u =  $1.6605402 \times 10^{-27} \text{ kg} \equiv 931.494 \text{ MeV}$  rest mass energy

$$1 \text{ eV} = 1.60217733 \times 10^{-19} \text{ J}$$

$$T_{1/2}(^{14}\text{C}) = 5730 \text{ years}$$

Ratio of carbon 14 to carbon 12 in the atmosphere,  $\frac{N(^{14}\text{C})}{N(^{12}\text{C})} = 1.2987 \times 10^{-12}$

Helium ( $^4\text{He}$ ) mass = 4.002603 u

Molybdenum ( $^{94}\text{Mo}$ ) mass = 93.905088 u

Ruthenium ( $^{98}\text{Ru}$ ) mass = 97.905287 u

Cerium ( $^{140}\text{Ce}$ ) mass = 139.905434 u

Neodymium ( $^{144}\text{Nd}$ ) mass = 143.910083 u

$$\lambda_{\max} = \frac{hc}{4.965kT}$$

$$\theta_{\min} = 1.22\lambda/D$$

$$I = \frac{2\pi\hbar c^2}{\lambda^5 \left( e^{\frac{\hbar c}{\lambda k}} - 1 \right)}$$

$$\int \cos^2 au \, du = \frac{u}{2} + \frac{\sin 2au}{4a}$$

$$\int \sin^2 au \, du = \frac{u}{2} - \frac{\sin 2au}{4a}$$