UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2008/2009

TITLE OF PAPER : CLASSICAL MECHANICS

COURSE NUMBER : P320

TIME ALLOWED : THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR QUESTIONS – EACH QUESTION CARRIES 25 MARKS – MARKS FOR EACH SECTION ARE SHOWN IN THE RIGHT-HAND COLUMN IN SQUARE BRACKETS

THIS PAPER HAS 11 PAGES INCLUDING THIS PAGE

A table of the values of the physical constants is provided

Question 1

Consider a light particle in orbit around a massive particle, which for convenience is placed at the origin of the co-ordinate system. Consider the motion of the light particle that orbits the massive particle – the force between the two particles is attractive and central. Two quantities, concerning the motion of the light particle are conserved: its energy (E) and its angular momentum (J);

$$\frac{1}{2}m\dot{\mathbf{r}}^2 + U(\mathbf{r}) = E \qquad (1)$$

$$m\mathbf{r} \times \dot{\mathbf{r}} = \mathbf{J} \qquad (2)$$

m is the mass of the light particle, $\mathbf{r}$ locates its position, $\dot{\mathbf{r}} \equiv \frac{dr}{dt}$ and U($\mathbf{r}$) is the potential energy.

The orbit of the light particle is planar. Why? [3]

It is sensible to introduce polar co-ordinates, r and θ, that lie in the plane of the orbit. θ is the angle of r relative to some arbitrary fixed direction at time t. Using these orthogonal co-ordinates, then

$$v^2 = \dot{r}^2 = v_r^2 + v_\theta^2.$$

Eliminate $\dot{\theta}$ from equations (1) and (2) and reproduce the "radial energy equation", namely

$$\frac{1}{2}m\dot{r}^2 + \frac{J^2}{2mr} + U(\mathbf{r}) = E. \qquad (\text{Here } \dot{r}^2 \equiv v_r^2)$$ [4]

The force acting between the light and massive particles is attractive and may be represented by

$\mathbf{F} = -\frac{k}{r^2}\hat{\mathbf{r}}$, where k is positive and $\hat{\mathbf{r}}$ is a unit vector in the direction of $\mathbf{r}$.

The "effective" potential, U_e, is

$$U_e(\mathbf{r}) = \frac{J^2}{2mr^2} + U(\mathbf{r}).$$

Show that U_e has a minimum at $r = \ell = \frac{J^2}{m|k|}$. [4]

(Reminder: $U(r) = -\int \mathbf{F}.d\mathbf{r}$).

Describe the orbit of the light particle when $E = -\frac{|k|}{2\ell}$. [6]

What is the eccentricity of this orbit? [2]

What is the ratio of the potential energy and the kinetic energy in such an orbit? [6]

Question 2

A famous experiment involving the scattering of α-particles (composed of 2 protons and 2 neutrons) by gold nuclei was performed by Geiger and Marsden at the suggestion of Rutherford in 1911. A stationary very thin foil of gold was the target, and a well-collimated beam of α-particles was directed at the foil.

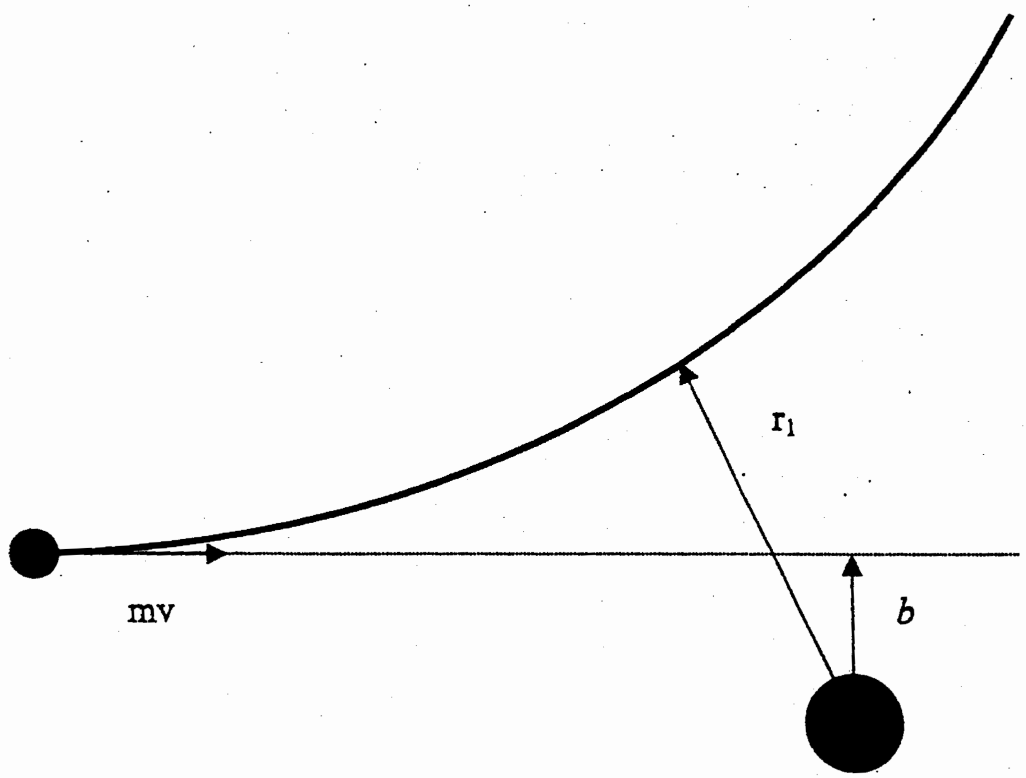
The experiment was conducted in an evacuated chamber and the gold foil needed to be very thin. Why? [2]

Describe the outcome of the experiment. [5]

Collisions between α-particles and the electrons surrounding a gold nucleus have very little effect on the trajectory followed by the α-particles. Explain why this is so. [3]

If there were no interaction between the α-particles and the gold nuclei then a particular α-particle would pass a distance b from a certain gold nucleus. b is known as the impact parameter. See the diagram. The trajectory of the α-particle is depicted as the heavy line in the diagram, the α-particle as the smaller black circle, and the gold nucleus as the larger black circle.

Derive an expression for the closest approach of an α-particle to a gold nucleus - r_1 in the diagram – in terms of parameters involving the interaction and the impact parameter. State any assumptions you make. [15]



Question 3

Galileo's principle of relativity states that

a) space is homogeneous
b) space is isotropic
c) time is homogeneous – that is, the measured time interval between two events is the same for all observers irrespective of the inertial frame they observe the two events from
d) velocities add.

Are any of these assumptions now believed to be incorrect? Give the label – the letter of the alphabet - for any that you believe to be incorrect. [4]

The Lagrangian, $L = T - U$, where T is the kinetic energy and U the potential energy, obeys the following

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}.$$

What do q and $\dot{q}_i$ represent? [2]

Using Galileo's principle of relativity and the fact that if the laws of physics are to be the same in different inertial frames then the Lagrangian must be the same in all inertial frames, show that three consequences are that one must have conservation of

a) energy [4]
b) linear momentum [4]
c) angular momentum [8]

What is an inertial frame? [3]