UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2008/2009

TITLE OF PAPER	**:**	**ELECTROMAGNETIC THEORY**
COURSE NUMBER	**:**	**P331**
TIME ALLOWED	**:**	**THREE HOURS**
INSTRUCTIONS		**ANSWER ANY FOUR QUESTIONS - EACH QUESTION CARRIES 25 MARKS – MARKS FOR EACH SECTION OF EACH QUESTION ARE SHOWN IN THE RIGHT-HAND COLUMN**

THIS PAPER HAS 10 PAGES INCLUDING THIS PAGE

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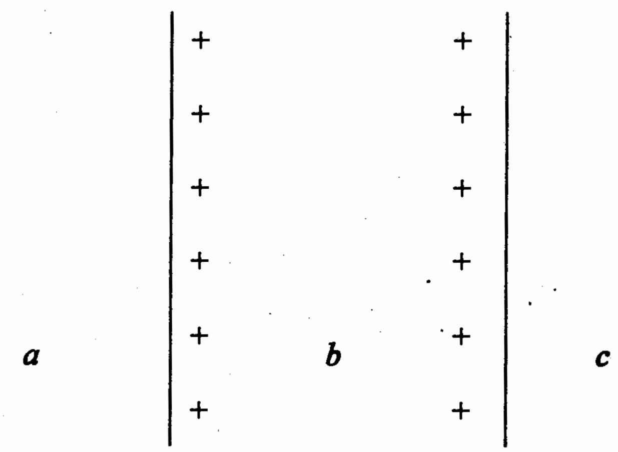
A table of the values of physical constants is provided in addition to a sheet of equations relevant to the course

Question 1

The diagram shows the distribution of charges in an infinite conducting sheet that has been given a net positive charge. σ is the charge density per unit area on both surfaces. Show that the electric field outside the conductor, at points *a* and *c*, is equal to $\frac{\sigma}{\varepsilon_0}$. [12.5]

Also show that the electric field within the interior of the conductor, at point *b*, is zero. [12.5]

[useful information and hints:
when a conducting plate is given a net charge, this charge distributes itself over the entire outer surfaces of the plate, and if the plate is of uniform thickness and is infinitely large, the charge per unit area is uniform and is the same on both surfaces. Hence the field of a charged, very large plate arises from the superposition of the fields from two sheets of charge. By symmetry the field is perpendicular to the plate, and if the plate is positively charged, it is directed away from the sheet. Construct suitable Gaussian "pill-boxes" to answer the question.]

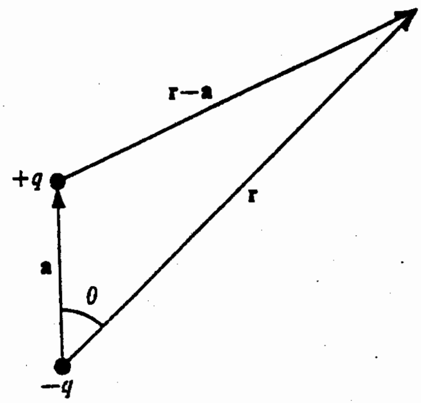


Question 2

An electric dipole consists of 2 equal and opposite charges, q and –q, placed close together. Let the distance that separates them be **a**, a vector which is parallel to the *z*-direction. One of the charges is placed at the origin of the co-ordinate system (-q). The potential at **r** is given by

$$V(\mathbf{r}) = \frac{q}{|\mathbf{r}-\mathbf{a}|} - \frac{q}{\mathbf{r}} \qquad \text{(see the diagram)}$$

From the diagram obtain an expression for $|\mathbf{r}-\mathbf{a}|^2$. This will involve the parameters r, a and $\cos\theta$. [5]



Evaluate an approximate expression for $\frac{1}{|r-a|}$. Assume that a<<r, so therefore $|r-a|$ can be expanded using the binomial expansion – see below. Since a<<r, there is no need to go beyond terms involving a^2 - terms in the expansion involving a^3 and higher orders may be neglected. [5]

In this approximate expression, a cosθ can be replaced by $\mathbf{a}.\hat{\mathbf{r}}$ where $\hat{\mathbf{r}}$ is a unit vector in the direction of **r**. Manipulate your approximate expression for $V(\mathbf{r})$ and replace a cosθ *with* $\boldsymbol{a}.\hat{\mathbf{r}}$ [5]

Keeping terms that are only linear in a and neglecting those involving a^2, and defining the electric dipole moment as $\mathbf{d} = q.\mathbf{a}$ show that

$$V(\mathbf{r}) = \frac{\mathbf{d}.\hat{\mathbf{r}}}{r^2} = \frac{d\cos\theta}{r^2}. \qquad [5]$$

Using spherical polar co-ordinates the electric field is given by

$$E_r = -\frac{\partial V}{\partial r}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

Show that the electric field produced by the electric dipole has cylindrical symmetry about the z–direction. [5]

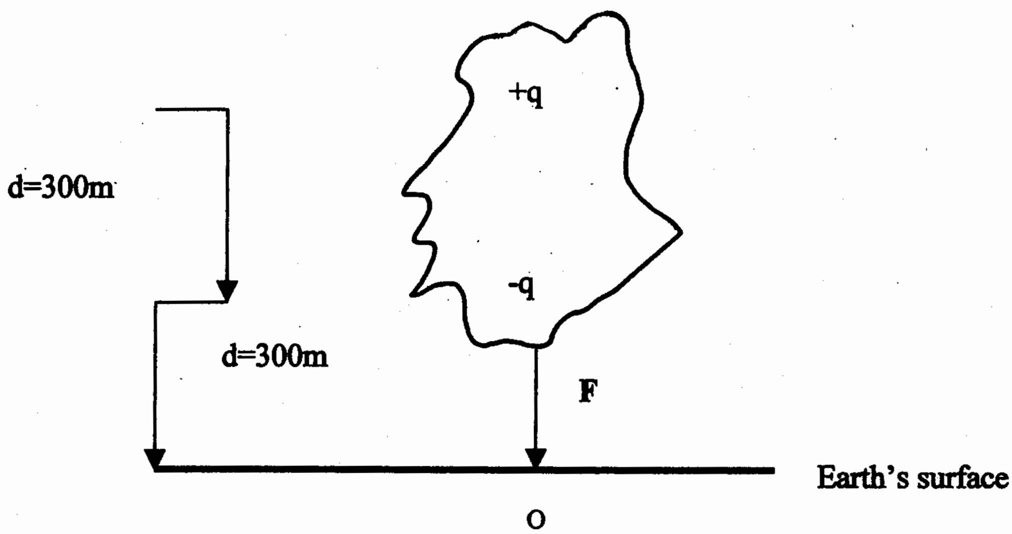
[the binomial theorem gives $(1-x)^n = 1 - nx + \frac{(n)(n-1)}{2!}x^2 + ...$, which is true for all n if $-1<x<1$]

Question 3

A cloud passed over PH2.1 on October 12th 2005. A vertical electric field of 100 volts/metre was recorded. The bottom of the cloud was at a height of 300 metres above the surface of the earth, and the top of the cloud was 300 metres above the bottom of the cloud. The cloud was electrically neutral but had a charge of +q at its top and –q at its bottom (see diagram). There were no other charges other than those on the cloud present in the atmosphere at the time.

Estimate the magnitude of the charge q and the external electric force (direction and magnitude) on the cloud. [25]

[hint: use the method of images – assume that the earth's surface is planar and flat, and is an equipotential]



Question 4

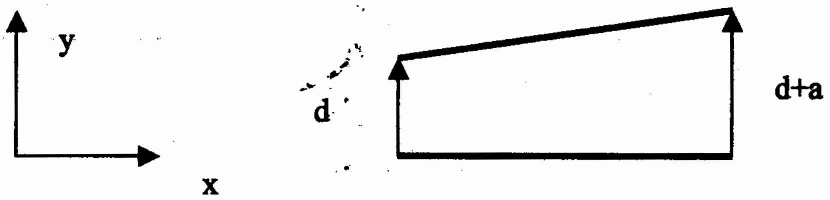
From manipulation of suitable equations from the sheet of equations, derive an expression for the phase and group velocity of light in free space. [15]

Comment on any attenuation of the wave. [5]

From expressions for $\boldsymbol{E}$ and $\boldsymbol{H}$ obtain the magnitude of the impedance of free space. [5]

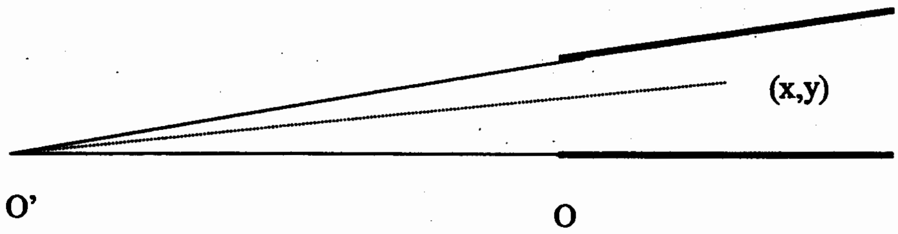
Question 5

You are given a badly-made parallel plate capacitor, The plates are flat, but not parallel to each other (see diagram).



The *z*-direction of the Cartesian axes used points into the plane of the diagram. Neglect edge effects – assume that the plates are very long in the *z*-direction. Therefore the electric field lies in the *xy*-plane. Determine the potential everywhere between the plates when a voltage difference of V is placed across the two conductors.

[hints: suppose that the intersection point of the extended lines of the two planes of the plates is at O'. See the diagram. Evaluate θ_0 and the length O'O.



Use cylindrical coordinates (r θ z') with the z' axis passing through O' and parallel to z. All planes perpendicular to the z-axis are equipotentials, so the potential inside the capacitor will depend only on θ and it satisfies

$$\Delta^2 V = \frac{1}{r^2}\frac{d^2 V}{d\theta^2} = 0$$

Question 6

A long thin rod (shown in the diagram) has a uniform distribution of charge on it, Suppose the total excess charge on the rod is Q. What will be the force experienced by a charge of q at a distance of a from one end of the rod? [25]

[hint: consider a small length of the rod, dx, which is at a distance x from q. What is the force experienced by q due to the charge within length dx? Integrate your expression from a to $(a + L)$.]