UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2008-2009

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) (i) Distinguish between a primitive unit cell and a conventional unit cell. (2 marks)

(ii) Draw conventional unit cells of face centred and body centred cubic lattices of lattice constant 'a'. For each lattice, write down the number of lattice points per cell and the volume of the primitive cell. (6 marks)

(iii) What is meant by packing fraction of a crystal? Determine the packing fraction of a bcc crystal (2+3 marks)

(b) (i) In the diagram of a cubic unit cell show a (110) and a (100) plane. (4 marks)

(ii) Calculate the separation between two (123) planes of an orthorhombic cell with a = 0.82 nm, b = 0.94nm and c = 0.75nm (3 marks)

(c) A first order reflection from the (111) planes of a cubic crystal was observed at a glancing angle of 11.2^0 when x-rays of wavelength 154 pm were used. Calculate the length of the side of each cell. (5 marks)

Question Two.

(a) (i) What is Van der Waals -London attractive interaction in inert gas crystals. (5 marks)

(ii) Explain how Pauli's exclusion principle is responsible for the repulsive interaction in inert gas crystals. (5 marks)

(b) (i) Derive the Bragg law $2d\sin\theta = n\lambda$ for diffraction of waves by a crystal lattice. (5 marks)

(ii) Explain why visible light cannot be used for Bragg reflection experiments. (2 marks)

(iii) In the X-ray photograph of a cubic lattice, lines are observed at the following Bragg angles in degrees: 6.6, 9.2,11.4 ,13.1, 14.7, 16.1, 18.6, 19.8. Assign Miller indices to these lines and identify the lattice type. (8 marks)

Question Three.

(a) Given below are that the translation vectors in the direct lattice and the reciprocal lattice respectively: **T** = n_1**a** + n_2**b**+ n_3**c** , **G** = h**A** + k**B**+ l**C**

(i) Write down vectors **A**, **B** and **C** in terms of **a**,**b** and **c**. (2 marks)

(ii) Show that exp (i**G**.**T**) = 1 (4 marks)

(b) (i) A wave of wave vector **k** is incident on a crystal specimen. The diffracted wave has wave vector **k'**. Show that diffraction condition for constructive interference between the two waves can be written as: **G** = **Δk**, where **Δk** = **k'** - **k** , where **G** is a reciprocal lattice vector.
What is the physical meaning of the above condition?

(10 marks)

Given: $n(\bar{r}) = \sum_{G} n_G \exp i\bar{G}.\bar{r}$

(ii) The geometric structure factor of a crystal is given below :

$S_G = \sum_{j=1}^{s} f_i \exp[-i2\pi(n_1 h + n_2 k + n_3 l)]$, where 's' is the number of atoms in the basis and n_1, n_2, n_3 are fractional coordinates. 'f' is the atomic form factor.

Explain the significance of this as regards the identification of lattice type using X-ray diffraction of crystals. Give bcc as an example. (9 marks)

Question Four.

(a) (i) Explain how an intrinsic sample of silicon can be made n -type or p - type by appropriate doping. (4 marks)

(ii) State what is meant by **effective density of states** of a semiconductor (2 marks)

(iii) The effective density of states in the conduction and valance bands of a semiconductor is given as:

$$N_{c,v} = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

Using the above expression write down the electron and hole concentrations in the conduction and valance bands. (2 marks)

(iv) show that the electrical conductivity of an intrinsic semiconductor can be expressed as

$\sigma_i = A\exp\left(\frac{-E_g}{2kT}\right)$ where E_g is its band gap. (4 marks)

(v) Explain how would you use this expression to find the band gap of a material experimentally. (4 marks)

(b) Calculate: (i) the effective density of states and
(ii) the intrinsic carrier concentration of silicon.

[Effective masses of electrons and hole are 1.1 m_0 and 0.56 m_0 respectively. Band gap of silicon = 1.1 eV]

(6 + 3 marks)

Question Five.

(a) Discuss briefly the free electron approximation in metals. (4 marks)

(b) Assume a plane wave $\psi_k(r) = \exp i(k.r)$, where symbols have the usual meanings, representing a free electron. Use the Schrodinger wave equation to obtain its energy eigenvalues ϵ_k. (4 marks)

(c) (i) What is meant by Fermi energy? (2 marks)

(ii) Use the results in (b) above to show how the Fermi energy is related to the electron concentration, and hence derive an expression for the density of states of the electrons in a metal. (10 marks)

(d) Calculate the Fermi energy of potassium given that it has a density of 8.6×10^2 kg m^{-3} and an atomic weight 39. (5 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27 \text{kg}}$
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}