UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

SUPPLEMENTARY EXAMINATION 2008/2009

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) (i) Draw ***one*** figure showing the conventional unit cell and its primitive unit cell in a face centred cubic (fcc) lattice of lattice constant 'a'. (4 marks)

(ii) Write down the translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ of the primitive cell of the fcc lattice in terms of its lattice constant 'a' (3 marks)

(iii) Find the number of lattice points in the conventional fcc unit cell. (2 marks)

(iv) Find the nearest neighbour distance of the fcc lattice. (show the working clearly) (4 marks)

(v) Calculate the packing efficiency of an fcc lattice. (4 marks)

(b) (i) The planes of a tetrahedron are $(111), (\bar{1}1\bar{1}), (1\bar{1}\bar{1}) \mathit{and} (\bar{1}\bar{1}1)$, Calculate the bonding angle. (4 marks)

(ii) Calculate the Bragg angle for second order reflection from (100) planes when 1.54 Å x-rays are incident on a cubic crystal lattice with lattice constant 4.0 Å. Calculate the energy in eV of these x-rays. (4 marks)

Question Two.

(a) (i) Explain ***ionic bonding*** in crystalline materials. (3 marks)

(ii) What is meant by the phrase ***Madelung energy of ionic crystals***? (3 marks)

(b) Derive an expression for the total lattice energy of a crystal having 2N ions at their equilibrium separation 'R_o'. (12 marks)

(c) A line of 2N ions of alternating charges (+/-) q have a repulsive potential energy of the form A/R^n, between nearest neighbours. Show that at equilibrium separation the potential energy,

$$U_{tot} = \frac{-2Nq^2 \ln 2}{R_0}\left(1 - \frac{1}{n}\right) .$$

[Given: Madelung constant = 2 ln2] (7 marks)

Question Three.

(a) State the basic assumptions of Einstein's theory of the specific heat of solids. (3 marks)

(b) Deduce an expression for the heat capacity of solids according to Einstein's theory.

[Given: the mean energy of a harmonic oscillator, $\bar{\varepsilon} = \hbar\omega\left(\frac{1}{2} + \frac{1}{e^{\hbar\omega/kT} - 1}\right)$]

(8 marks)

(c) Show that in the high temperature limit (i.e. $T > \hbar\omega/k$) , Einstein's theory agrees with the classical Dulong-Petit law.

[Given that: for small values of x, $e^x \approx 1 + \frac{x}{1!} + \frac{x^2}{2!}$]

(7 marks)

(d) State how, at low temperatures, the specific heat of a solid varies with temperature and discuss how far Einstein's theory agrees with it. (7 marks)

Question Four.

(a) (i) Use the Schrodinger wave equation to show how the energy of free electron varies with its wave vector.

(6 marks)

(ii) Sketch a plot of energy E versus the wave vector **k** for a free electron.

(3 marks)

(b) (i) According to Kronig-Penny model, energy-wave vector relation for an electron in a periodic potential can be written as:

$$P\frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

where 'α' is a function of energy and 'a' is the width of the potential well.

Take P = (3/2)π, and for various values of αa, (π/2,π,3π/2, 2π, 5π/2, 3π, 7π/2 , 4π etc), obtain the LHS of the above expression and sketch a graph against αa.

(10 marks)

(ii) Sketch a plot of energy versus wave vector **k** for an electron in a periodic potential based on observations from the sketch in (b) (i) above and comment.

(6 marks)

Question Five.

(a) (i) State the assumptions made by Drude in his classical theory of electron gas

(3 marks)

(ii) Show that according to Drude theory the conductivity of a metal can be expressed as:

$$\sigma = \frac{ne^2\tau}{m}$$

Where symbols have their usual meanings. (10 marks)

(iii) Define the following terms as regards the motion of an electron in a solid:
1. Mean free path
2. Mobility (4 marks)

(iv) Show how the conductivity of metal obtained in question (ii) above can be expressed in terms of the mobility of electrons.

(4 marks)

(b) Given that the intrinsic carrier concentration of silicon is $1.48 \times 10^{16}\ m^{-3}$, calculate its conductivity at 300K

[electron, hole mobilities are $0.14\ m^2 / Vs$ and $0.05\ m^2 / Vs$ respectively]

(4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$He atom		6.65×10^{-27} kg
Mass of $_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}