UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2008/2009

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

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THIS PAPER CONTAINS FIVE QUESTIONS

Question One.

(a) State what is meant by the following terms for a system of particles.

(i) Macrostate
(ii) Microstate
(iii) Statistical weight

(6 marks)

(b) A system has 5 classical particles to be arranged in 3 energy levels. Using appropriate expression find the weights and hence the most probable distribution of these particles for the following cases:

(i) The energy levels are non-degenerate.

(ii) The energy levels are doubly degenerate

(12 marks)

(c) (i) What is meant by Phase space? (2 marks)

(ii) Derive expressions for the volume element in phase space in terms of

1. Momentum p

2. Energy ϵ. (3 + 4 marks)

Question Two.

(a) (i) Distinguish between an extensive and an intensive variable used in thermodynamics. Give at least one example each. (4 marks)

(ii) Write down the equation that links entropy of a system with its thermodynamic probability. (2 marks)

(iii) Two systems of identical particles 1 and 2 have entropies S_1 and S_2 and statistical weights W_1 and W_2. When the two systems are mixed together, what is:

1. The total entropy S_T (2 marks)

2. The total statistical weight W_T of the mixture? (2 marks)

(iv) Do these results agree with your equation in (ii) above? Explain. (3 marks)

$$\text{Given: } W = N!\prod_s \left\{\frac{g_s^{n_s}}{n_s!}\right\}$$

(b) Derive an expression for the entropy 'S' of a system in terms of its partition function 'Z'.

(8 marks)

(c) Show that the partition function of a perfect classical gas can be expressed as:

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

(4 marks)

[see appendix for definite integrals]

Question Three.

(a) Given that a one-dimensional harmonic oscillator has discrete energy given by $\varepsilon = \left(n + \frac{1}{2}\right)h\nu$, where symbols have their usual meanings, obtain an expression for its mean energy. (9 marks)

Given: mean energy $= kT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V$

(b) Assume that a solid has N atoms each having three mutually independent vibrations. Using your results in question (a) above, obtain an expression to show how the specific heat capacity of the solid varies with temperature T.

(8 marks)

(c) Show that at high temperatures specific heat capacity of the solid is equal to the classical value 3Nk. (4 marks)

(d) Experimentally it is observed that at low temperatures specific heat of solids varies proportional to T^3. Does the above theory agree with this? Comment.

(4 marks)

Question Four.

(a) Use the Bose-Einstein distribution function of an assembly of identical non-interacting particles in thermal equilibrium to derive the Planck's radiation law for spectral distribution of energy radiated from a constant temperature enclosure.
(9 marks)

(b) Obtain an expression for the total energy per unit volume emitted from the enclosure at temperature T. (5 marks)

(c) (i) State briefly what is ***Bose_Einstein condensation.*** (3 marks)

(ii) The density of ideal gas consisting of particles having mass 6.65×10^{-27} kg is $1.17 \times 10^{26}\ m^{-3}$.

1. Calculate the Bose temperature T_B of the gas. (5 marks)

2. What fraction of the particles will be in the ground state at a temperature of $0.1T_B$. (3 marks)

Given:

$$N = 2.612V\left(\frac{2\pi mkT_B}{h^2}\right)^{3/2}$$

Question Five.

(a) The Fermi function of system is given as: $f(\varepsilon) = \frac{1}{e^{(\varepsilon-\varepsilon_F)/kT} + 1}$ where symbols have their usual meanings. Obtain its values at absolute zero temperature, for the cases $\epsilon > \epsilon_F$ and $\epsilon < \epsilon_F$.

What is the physical meaning of the these results? (7 marks)

(b) The Fermi level of a solid 8.6 eV. Find the probability of occupation of electron:

(i) in an energy level 0.1 eV above the Fermi level at 300K and at 400K

(ii) in an energy level 1.0 eV above the Fermi level at 300K and at 400K

Comment on the results. (6 marks)

(c) Use Fermi-Dirac statistics to show that the contribution by free electrons in a metal toward its specific heat is proportional to the absolute temperature.

Given: Fermi energy $\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$

(12 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27 kg}$
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}