UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2008_09

TITLE OF THE PAPER: **COMPUTATIONAL PHYSICS-II**

COURSE NUMBER : **P482**

TIME ALLOWED : **SECTION A: ONE HOUR.**
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

SECTION A **: THIS IS WRITTEN PART ON YOUR ANSWER BOOK.**
CARRIES A TOTAL OF 30 MARKS.
SECTION B**: THIS IS A PRACTICAL PART WHICH YOU WILL**
WORK ON A PC AND SUBMIT THE PRINTED OUTPUT.
CARRIES A TOTAL OF 70 MARKS.

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED **APPENDIX** WHEN NECESSARY.

THIS PAPER HAS FOUR PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Instruction: Use the information given in Appendix when necessary.

Q.1.

(i) Write pseudo-code to calculate an integral of the form (6)

$\int_a^b f(x)dx$ using Monte-Carlo method. Discuss how to estimate the error involved in the calculation. State the advantages and disadvantages in the use of Monte-Carlo method.

(ii) Convert the pseudo-code into a procedure program in Maple. (9)

Q.2. Write a procedure to solve a differential equation of the form (15)

$$a\frac{d^2y(x)}{dx^2}+b\frac{dy(x)}{dx}+c\,f(x)=0$$

with initial conditions $y(0)=\alpha$ and $\left.\frac{dy(x)}{dx}\right|_{x=0}=\beta$. using Euler method.

Note: You have to convert the second order differential equation into two first order differential equations in order to use the Euler method.

Q.3.

(i) Write a short note on the Random Numbers and its use in practical applications. (3)

(ii) Consider a walk restricted to walk only along East or West direction Assume that the walker can take a step of length ***S*** in either direction with equal probability.

(a) Write a pseudo code to calculate the distance travelled by the walk in N number of steps. (4)

(b) Convert the pseudo code into a program in Maple. (8)

Note: `stats[random,uniform](n):`
`# produces n uniform random numbers in the range (0,1).`

SECTION B
(Practical Section)

Instruction: Use the information given in Appendix when necessary.

Q.4 A liquid of low viscosity, such as water, flows from an inverted conical tank with circular orifice at the rate

$$\frac{dx}{dt} = -0.6\pi r^2 \sqrt{-2g}\,\frac{\sqrt{x}}{A(x)}$$

where r is the radius of the orifice, x is the height of the liquid level from the vertex of the cone, and A(x) is the area of the cross section of the tank x units above the orifice. Initial water level of the tank is 2m .

Given: r =0.02 m, g=-9.8 ms^{-2} and $A(x)=0.14\,\pi\,x^2$.

(i) .Use dsolve to get the analytic solution for x(t). (5)
(ii) Plot the solution for t=0..300s (2)
(iii) Calculate the time when the tank gets empty. (10)
(iv) Write a program to compute X(t) as a function of time for t=0..5s.with increment of time Δt=1.0s using RK-4^{th} order method. Plot obtained output (15)
(v) Plot the two solutions on the same graph. How does the numerical calculation compare with exact calculation? Comment. (3)

Q.5. Consider random walk problem in two dimensions. The walk starts at the origin which is at a lattice point .Assume length of each step to be of one unit and only directions available for the walk are :

East, South and **North.**

Each point on the lattice is recognized by the coordinates ***(x,y)*** from the origin .

(i) Write a procedure to find the co-ordinate ***(x,y)*** of the walker after ***N*** number of steps. using the random number generator of uniform distribution of Maple . Include in your procedure a global variable for the seed used in generating the random numbers. (15)
(ii). Use the procedure to calculate $R=\sqrt{x^2+y^2}$ the distance travelled by the walker in 1000 steps. (5)
(iii) Use the procedure to calculate the distance ***R*** for ***N=1000 to 10000*** steps with an increment of ***ΔN=1000***. Treat each walk of ***N*** steps as an independent walk. (12)
(iv) What do you conclude from the output. (3)

Note: `stats[random,uniform](n):   # produces n uniform random numbers in the range (0,1).`

@@@@END OF EXAMINATION@@@@

Appendix:

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x,y)$ with the given initial boundary condition $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h\,f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = h\,f(x_i, y_i)$$
$$k_2 = h\,f(x_i + 0.5\,h,\ y_i + 0.5\,k_1)$$
$$k_3 = h\,f(x_i + 0.5\,h,\ y_i + 0.5\,k_2)$$
$$k_4 = h\,f(x_i + h,\ y_i + k_3) \qquad \text{and} \qquad h = x_{i+1} - x_i$$

2. Monte Carlo Evaluation of Integrals:

Integral of the form $F = \int_a^b f(x)dx$ is given by

$$F = \frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i)$$

where x_i is the i^{th} random number of n random numbers distributed uniformly in the interval (a,b). The standard deviation can be estimated from the points sampled in evaluating the integral by

$$\sigma_n = \left[\frac{\frac{1}{n}\sum_{i=1}^{n}[f(x_i)]^2 - [\frac{1}{n}\sum_{i=1}^{n} f(x_i)]^2}{n-1}\right]^{1/2}$$

with 68.3% confidence.