UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMETARY EXAMINATION 2008_09

TITLE OF THE PAPER: **COMPUTATIONAL METHODS-II**

COURSE NUMBER : **P482**

TIME ALLOWED : **SECTION A: ONE HOUR.**
SECTION B: TWO HOURS.

INSTRUCTIONS:

THERE ARE TWO SECTIONS IN THIS PAPER:

SECTION A **: THIS IS WRITTEN PART ON YOUR ANSWER BOOK. CARRIES A TOTAL OF <u>30</u> MARKS.**
SECTION B**: THIS IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. CARRIES A TOTAL OF <u>70</u> MARKS.**

ANSWER **ANY TWO** QUESTIONS FROM **SECTION A** AND **ALL** THE QUESTIONS FROM **SECTION B.**

MARKS FOR EACH QUESTION ARE SHOWN IN THE RIGHT-HAND MARGIN.

USE THE INFORMATION GIVEN IN THE ATTACHED **APPENDIX** WHEN NECESSARY.

THIS PAPER HAS <u>FOUR</u> PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THE PAPER UNTIL THE INVIGILATOR HAS GIVEN PERMISSION.

SECTION A
(Written Section)

Instruction: Use the information given in Appendix when necessary.

Q.1. Write short notes on

(i) Random numbers generated on a computer. [5]
(ii) Method of Monte Carlo Integration. [5]
(iii) Fourier transform and fast Fourier transform. [5]

Q.2. (a) A multiple integral of the form $I = \iint_R f(x,y)\,\rho(x,y)dxdy$ where the region $R = \{(x,y) | a \le x \le b, c \le y \le d\}$, can be calculated by use of Monte Carlo method and the algorithm for the estimation of the integral is

$$I = \frac{(b-a)(d-c)}{n}\sum_{i=1}^{n} f(x_i,y_i)\,\rho(x_i,y_i)$$

Here n random numbers for x_i *and* y_i are generated ***independently*** in the interval $a \le x \le b$ *and* $c \le y \le d$.

Write a psuedo code for calculating I. [5]

(b) A plane lamina is defined to be a thin sheet of continuously distributed mass. If $\rho(x,y)$ is the function describing the density of a lamina having a shape of a region R in the xy-plane, then the center of mass of the lamina $(\bar{x},\bar{y})$ is defined by

$$\bar{x} = \frac{\iint_R x\,\rho(x,y)dxdy}{\iint_R \rho(x,y)dxdy}, \quad \bar{y} = \frac{\iint_R y\,\rho(x,y)dxdy}{\iint_R \rho(x,y)dxdy}$$

Write a program to calculate $(\bar{x},\bar{y})$ using Monte-Carlo method with the density function $\rho(x,y) = e^{-(x^2+y^2)}$ and the region described by $0 \le x \le 1.0$ *and* $0 \le y \le 1.0$. Use 1000 random numbers. [10]

Note: **stats[random,uniform](n):**
 # produces n uniform random numbers in the range (0,1).

Q.3. The equation of motion for the driven oscillator with damping force is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

where k is the force constant, b is damping constant, and F_0 amplitude of driving force of angular velocity ω.

Assume initial conditions to be at *t=0* , *x(t=0)=0* and $\left.\frac{dx}{dt}\right|_{t=0} = 0$.

(a) Convert the given equation into two first order differential equations. [2]

(b) Write an algorithm in pseudo code to find the solution x(t) on the interval $0 \le t \le 25$ at n=200 points using a numerical method of your choice. [10]

(c) Extend the above pseudo code for n to n+10 and verify if the integral is convergent with a precision of 0.01. [3]

SECTION B
(Practical Section)

Instruction: Use the information given in Appendix when necessary.

Q.4 Two dimensional trajectory of a projectile of mass m, in the presence of air resistance is given by the following equations:

$$m\frac{dv_x}{dt} = -f_d\cos(\vartheta) \quad , \quad m\frac{dv_y}{dt} = -mg - f_d\sin(\vartheta)$$

with

$$\frac{dx}{dt} = v_x \quad , \quad \frac{dy}{dt} = v_y$$

Here ϑ = the angle at which the projectile is directed above the horizontal.

v_x= horizontal component of velocity of the projectile.

v_y= vertical component of velocity of the projectile.

f_d =force due to air resistance= $k\ v^2$.

where k= co-efficient of air resistance and $v^2=v_x^2+v_y^2$.

g=acceleration due to gravity=9.8 .

Using the Euler method of solving first order differential equations, we can write the four equations as

$$x(i+1) = x(i) + v_x(i)\,\Delta t$$
$$v_x(i+1) = v_x(i) - \Delta t\, k(v_x(i)^2 + v_y(i)^2)\cos\vartheta / m$$
$$y(i+1) = y(i) + v_y(i)\,\Delta t$$
$$v_y(i+1) = v_y(i) - g\Delta t - \Delta t\, k(v_x(i)^2 + v_y(i)^2)\sin\vartheta / m$$

Given m=7, k=0.01.

Initial conditions are: time t=0 , x(0)=0,y(0)=0,

$v_x(0)$= 30 cos(θ) . $v_x(0)$= 30 sin(θ) .

(i) Write a program to calculate at launching angle θ= 30^0 , x,y,v_x and v_y starting at time t=0 to 10 s at intervals of Δt=0.1 s and determine the range of the projectile. [15]

(ii) Convert the program into a procedure which can calculate range for any given launching angle θ. [10]

(iii) Test the procedure for the angles θ=20^0 , 30^0 , 40^0, 50^0. [8]

(iV) From the result of (iii), predict the approximate angle at which the range is maximum. [2]

Note:

1. The range is determined by x(i) when y(i) is negative. i=0,1,2,......n
where n=10/0.1=100 in this case.
All the data given is in SI units as applicable in each individual cases.

2. In using the functions sin or cos the angle has to be in radians.

Q.5. Consider a two dimensional lattice with 200 lattice points along the x-direction and 200 lattice points along the y-direction forming a set of square lattices. The distance between lattice points is of unit length. The total number of lattice points is 200 x 200 . [35]

Assume that all the lattice points at (x ,y=20) and (x ,y=-20) for all x have reflecting property and all the lattice points at (x=20, y) and (x=-20, y) have a sink (a point of no return). That is if a walker reaches y=20 the next step is towards south or if the walker reaches y=-20, the next step is towards north.

Here we have assumed y-direction to be north-south. On the other hand, if a walker reaches x=20 or x=-20 for any y, a point of no return is encountered (that is the walk comes to an end).

Write a program and execute it to find the number of steps a walker would take before encountering the point of no return in x-direction. Assume that the random walk starts from (x=0,y=0).

Use the uniform random number generator available in Maple.

Note: **stats[random,uniform](n): # produces n uniform random numbers in the range (0,1).**

@@@@END OF EXAMINATION@@@@

Appendix:

1. Solution of First Order Differential Equation with initial Conditions:

The equation is of the form $\frac{dy}{dx} = f(x,y)$ with the given initial boundary condition $y(x_0) = \alpha$.

(i) Euler's Method:

$$y_{i+1} = y_i + h\, f(x_i, y_i) \qquad \text{where } h = x_{i+1} - x_i$$

(ii) Fourth Order RK-Method:

$$y_{i+1} = y_i + (k_1 + 2k_2 + 2k_3 + k_4)/6$$

where

$$k_1 = h\, f(x_i, y_i)$$
$$k_2 = h\, f(x_i + 0.5\,h,\ y_i + 0.5\,k_1)$$
$$k_3 = h\, f(x_i + 0.5\,h,\ y_i + 0.5\,k_2)$$
$$k_4 = h\, f(x_i + h,\ y_i + k_3) \qquad \text{and} \quad h = x_{i+1} - x_i$$