UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2009/10

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Body 1 is projected vertically upward with a velocity υ_0. At a later time T_1 a second body, Body 2 is also projected vertically upward from the same point. The two bodies collide in mid air at a height h a time T_2 after the launching of the second body. Make an illustrative diagram and determine the two equations that can enable you to find the height h in terms of T_1, T_2 and g. Do not solve for h. **(7 marks)**

(b) A particle moves in an Archimedean spiral (Figure 1.) given by the equation

$$\vec{r} = b\omega t\,\hat{r},$$

where b is a constant, ω is a constant angular velocity, and t is the time. The plain polar coordinate unit vectors are

$$\hat{\theta} = -\sin(\omega t)\hat{i} + \cos(\omega t)\hat{j} \text{ and}$$

$$\hat{r} = \cos(\omega t)i + \sin(\omega t)\hat{j}.$$

(i) Find the velocity of the particle as a function of time. **(2 marks)**

(ii) Write the velocity obtained in (i) in plane polar coordinates. And state what you understand by each component of this motion. **(2 marks)**

(iii) Show the directions of the components of the velocity in the spiral path. **(2 marks)**

(iv) Find the acceleration of the body as a function of time. **(3 marks)**

(v) Write the acceleration in plane polar coordinates and describe the terms obtained in the solution. **(2 marks)**

(vi) Illustrate the directions of the components of the acceleration in the spiral path. **(2 amrks)**

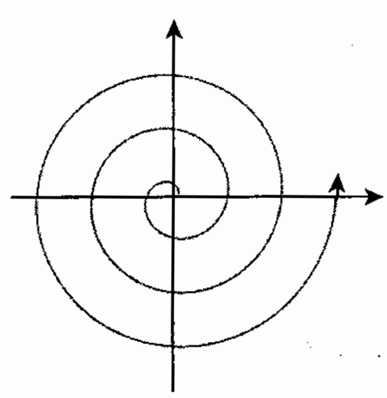

Figure 1.

(c) Use spherical coordinates to find the volume of quarter hollow hemisphere of inner radius R_1 and outer radius R_2. **(5 marks)**

QUESTION 2

(a) What horizontal force F must be applied to the cart of mass M in Figure 2 for the block of mass m_1 and m_2 to remain stationary relative to the cart? The coefficient of static friction between m_1 and the cart is μ. The mass of the pulley and its friction are negligible. The friction between the wheels and the floor is also negligible. Your answer must be in terms of M, m_1, m_2, g and μ. Start by making force diagrams for m_1 and m_2. **(8 marks)**

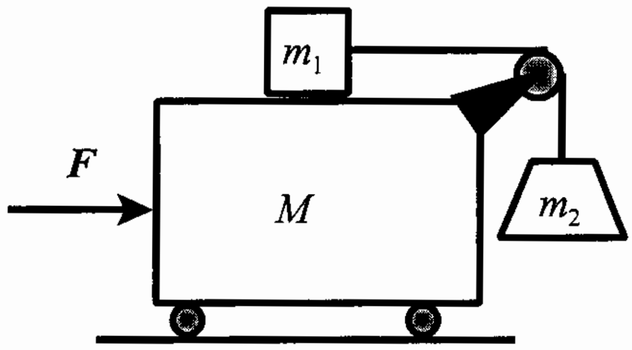

Figure 2.

(b) A particle of mass m is attached to a fixed point A on a vertical shaft by means of a string of length $5l$. The particle is attached to one end of a rod of length $5l$ and negligible mass. The other end of the rod is smoothly joined to a point B on the shaft which is $6l$ vertically below A. See Figure 3. The particle rotates in a horizontal circle with angular velocity $\omega = (g/10l)^{½}$ rad/s.

(i) Make a resolved force diagram for the mass m to solve (ii). **(5 marks).**
(ii) Write down the equations of motion for the mass to enable you do (iii). **(3 marks)**
(iii) Determine the tension in the string and the thrust in the rod. **(9 marks)**

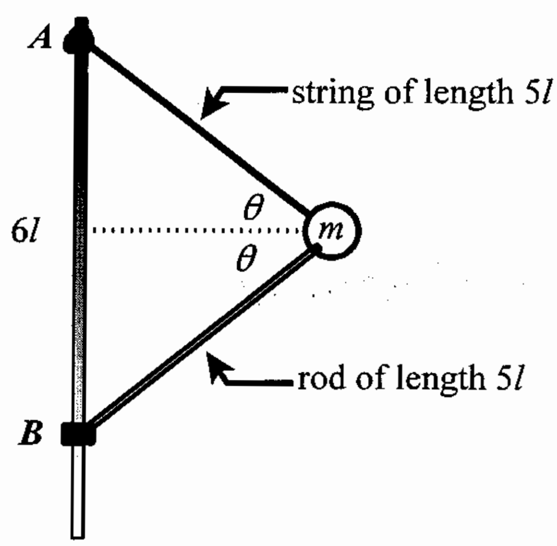

Figure 3.

QUESTION 3

(a) Find the centre of mass of a cone of base radius a and height h. The volume of a cone is $V = \frac{1}{3}\pi a^2 h.$ **(15 marks)**

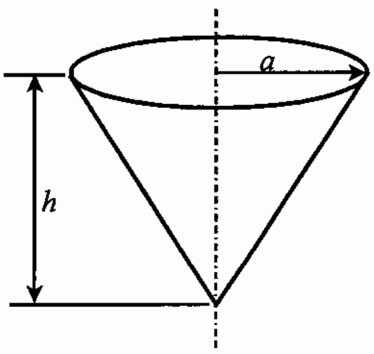

Figure 4.

(b) A rocket of mass $M = 4m$ is projected at an angle θ with the horizontal. At the highest point of its trajectory it breaks into three parts. A part of mass m falls (with zero initial velocity) directly below the point of explosion which is a distance L from the launching point. A second piece of mass m follows the original path of the trajectory. The other piece of mass $2m$ lands somewhere else.

(i) Make a sketch that illustrates the landing points of all the three masses. Give a brief discussion on the basis of momentum, that can enable you to determine the landing point of the third mass. **(5 marks)**

(ii) Determine where the piece of mass $2m$ lands. **(5 marks)**

QUESTION 4

(a) A particle of mass m is attached to point O by an inextensible string of length l and negligible mass. The mass hangs below point O, when it is given an instantaneous horizontal velocity

$$u = 2\sqrt{gl},$$

where g is the gravitational acceleration. See Figure 5.

(i) At what angle θ does the mass deviate from a circular path? **(12 marks)**

(ii) What is the minimum initial velocity u to enable the mass to complete full circles in its motion? **(5 marks)**

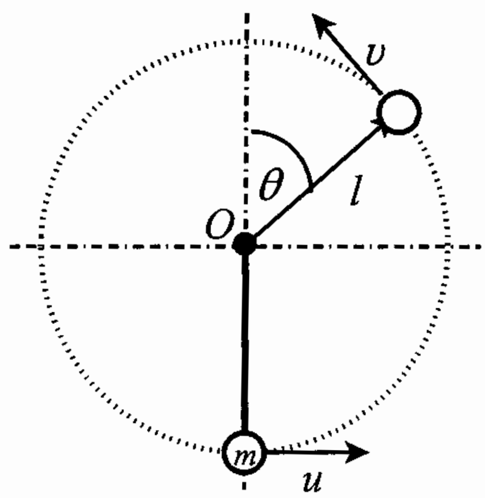

Figure 5.

(b) Consider a mass attached to a spring on a frictionless surface. Show that such a mass undergo simple harmonic motion when disturbed by a distance x_0 from the equilibrium point. The force on the mass if $F = -kx$ and the velocity at x_0 is $v_0 = 0$. **(8 marks)**

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) = \sin^{-1}\left(\frac{x}{a}\right).$$

QUESTION 5

(a) Find the moment of inertia of a hemisphere of mass M and radius R about an axis AA$'$ as shown in Figure 6. The volume of a sphere is $V_s = \frac{4}{3}\pi R^3$. **(10 marks)**

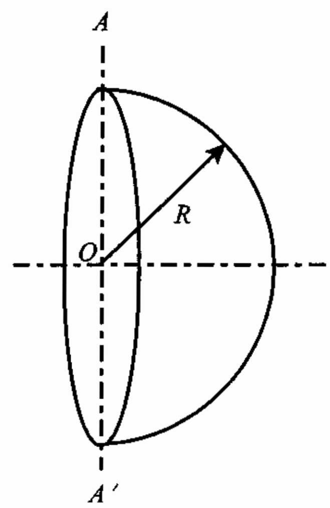

Figure 6.

(b) A uniform drum of radius b and mass M rolls without slipping down a plane inclined at angle θ. Find the acceleration along the plane taking the moment of inertia of the drum to be $I_0 = Mb^2/2$. Note, there is friction between the drum and the plane which results in the rotation. **(8 marks)**

(c) Derive the work energy theorem associated with rotational motion of a rigid body. **(7 marks)**