UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2009/10

TITLE OF PAPER:	MECHANICS
COURSE NUMBER:	P211
TIME ALLOWED:	THREE HOURS
INSTRUCTIONS:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
	EACH QUESTION CARRIES 25 MARKS
	MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Derive the two kinematic equations:

$v = v_0 + at$, and **(3 marks)**

$x = x_0 + v_0 t + 1/2at^2$ **(3 marks)**

(b) Body 1 is dropped vertically downward with zero initial velocity from a height h. At the same time a second body, Body 2 is projected vertically upward with initial velocity v_0. The two bodies collide in mid air at some height h' a time T later. Make an illustrative diagram and determine the gravitational acceleration g in terms of h, T and v_0. **(10 marks)**

(c) A tyre of radius R rolls in a straight line without slipping. Its centre moves with constant speed V. A small stone is lodged at its circumference and touches the origin at $t = 0$ s. (See Figure 1).

(i) What is the stone's position $\vec{r}$ as a function of time? **(3 marks)**

(ii) What is the acceleration of the stone as a function of time? **(3 marks)**

(iii) What is the acceleration of the stone as a function of time? **(3 marks)**

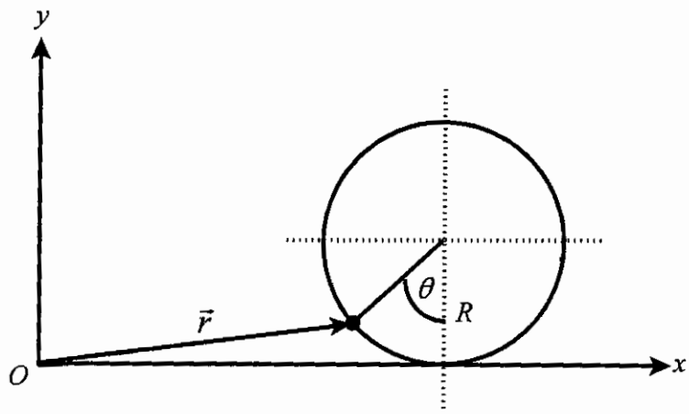

Figure 1.

QUESTION 2

(a) Consider the system shown in Figure 2. The pulleys are considered to be frictionless and with negligible masses. The mass of the string is also negligible. If the system is released from rest, how far does it go in time t? **(6 marks)**

(b) A body of mass M is hung from the roof of an elevator which accelerates with an acceleration a upward. Find an expression for the tension in the string in terms of a, M, and the acceleration due to gravity g. **(3 marks)**

(c) An automobile enters a turn (Figure 2) whose radius is R. The road is banked at an angle θ, and the coefficient of friction between the wheels and the road is μ.

(i) Make a force diagram for the vehicle when it is moving very slowly. **(3 marks)**

(ii) Find the minimum speed for the vehicle to stay on the road without skidding down the incline in terms of μ, R, g and θ. **(5 marks)**

(iii) Make a force diagram for the vehicle whern it is moving very fast. **(3 marks)**

(iv) Find the maximum speed for the vehicle to stay on the road without skidding up the incline in terms of μ, R, g and θ.. **(5 marks)**

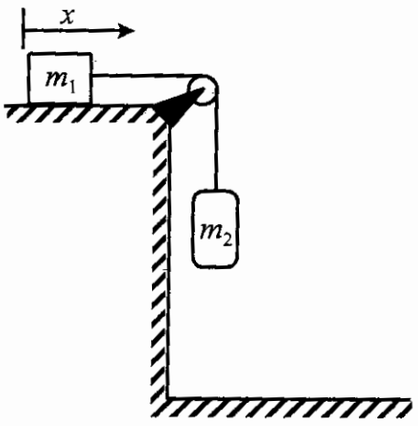

Figure 2.

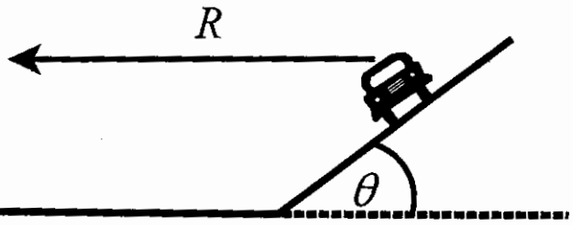

Figure 3

QUESTION 3

(a) A cone of base radius a and height h stands on its apex as shown in Figure 4. A cone is part of a cylinder. The cylindrical coordinates are ρ, ϕ and z. In this problem you want to set up the equations that can enable you to find the centre of mass of the cone. The density of the cone is D.

(i) Write down the equation for obtaining the centre of mass of a continuos body. **(3 marks)**

(ii) With the help of Figure 5. Determine the area element dA in cylindrical coordinates. **(3 marks)**

(iii) Write down the volume element dV in cylindrical coordinates. **(2 marks)**

(iv) Write down the mass element dm in cylindrical coordinates. **(1 mark)**

(v) Write down the vector $\vec{r}$ that locates the mass element dm in cylindrical coordinates in terms of its Cartesian counterparts. **(4 marks)**

(vi) Write down the equation you would use to determine the centre of mass of the cone and Show qualitatively where the centre of mass of the cone lies. **(4 marks)**

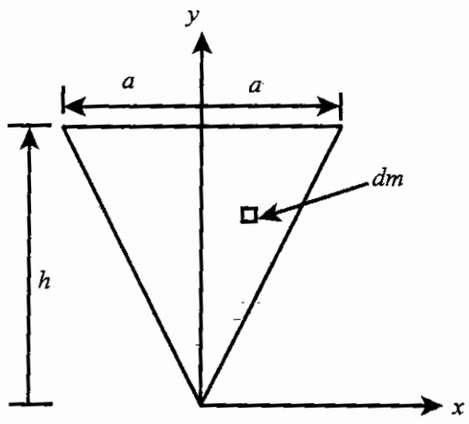

Figure 4.

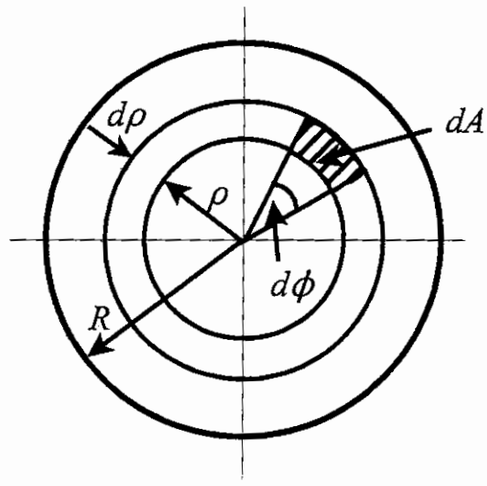

Figure 5.

(b) An instrument-carrying projectile accidentally explodes at the top of its trajectory. The horizontal distance between the launch point and the explosion point is L. The projectile breaks into two pieces which fly apart horizontally after the explosion. The larger piece has mass $3m$ and the smaller piece has mass m. The smaller piece returns to the launching point.

(i) Make a sketch that illustrates the landing points of the two pieces. **(3 marks)**

(ii) Determine where the larger piece land from the launching point? **(5 marks)**

QUESTION 4

(a) A particle moves from the highest point of a smooth sphere of radius R with an initial velocity u. See Figure 6.

(i) Determine the normal force on the particle while it moves on the sphere in terms of u, R, g and θ. Hint: Use the work-energy theorem. **(9 marks)**

(ii) Find an expression for the angle θ at which the particle leaves the sphere and evaluate the angle θ for $u = \sqrt{\frac{Rg}{2}}$. **(5 marks)**

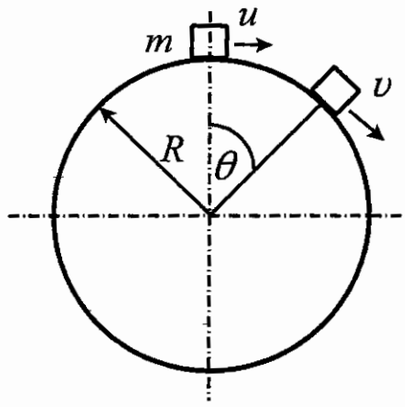

Figure 6.

(b) A potential energy function on a particle of mass m is given by $U = 3Ax^3 - 72Bx$, where A and B are positive constants with appropriate units.

(i) Find the force acting on the body. **(3 marks)**

(ii) Find the equilibrium points. **(2 marks)**

(iii) Determine the stability of the equilibrium points. **(3 marks)**

(iv) Find the frequency of small oscillations about the equilibrium points. **(3 marks)**

QUESTION 5

(a) (In this problem α, M, υ, R and ω are constants, t is time and g is the gravitational acceleration). A body of mass M has a momentum $\vec{p} = m\upsilon(\cos\omega t\hat{i} + \sin\omega t\hat{j})$, and is acted upon by a force

$$\vec{F} = \frac{Mg\sin\alpha}{\cos\alpha}\hat{r} = -Mg\tan\alpha(\cos\omega t\hat{i} + \sin\omega t\hat{j}).$$

(i) Find the angular momentum and torque if

$$\vec{r} = R(\cos\omega t\hat{i} + \sin\omega t\hat{j}).$$

(6 marks)

(ii) Find the angular momentum and torque if

$$\vec{r} = R(\cos\omega t\hat{i} + \sin\omega t\hat{j}) - l\cos\alpha\hat{k}.$$ **(9 marks)**

(b) Find the moment of inertia of a uniform disk of mass M, radius R about its axis of symmetry. **(8 marks)**

(c) How does a physical pendulum differ from a simple pendulum? **(2 marks)**