UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2009/2010

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:
Answer any four Questions.
Each Question carries 25 Marks.
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Question 1

(a) If there are more electric field lines leaving a Gaussian surface than there are entering the surface, what can you conclude about the net charge enclosed by that surface.

[2 marks]

(b) If the charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss' law to find the electric field? Explain.

[3 marks]

(c) A uniform electric field exists in a region of space in which there are no charges. What can you conclude about the net electric flux through a Gaussion surface placed in this region of space. Explain

[2 marks]

(d) A uniform electric field $n\hat{x} + m\hat{y}$ intersects a surface of area A. What is the flux through this area if the surface lies

(i) in the yz plane?

[3 marks]

(ii) in the xz plane?

[3 marks]

(iii) in the xy plane?

[3 marks]

(e) A thin spherical shell of radius a has a charge $+Q$ evenly distributed over its surface.

(i) Find the electric field at a distance r from the center of the shell, where $r < a$, and $r > a$

[6 marks]

(ii) Sketch electric field E as a function of r.

[3 marks]

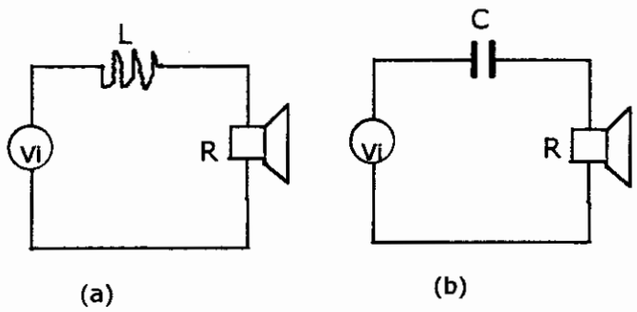

FIG. 1:

Question 2

(a) A RLC circuit consists of a 160Ω resistor, a $21\mu F$ capacitor and a $460mH$ inductor, connected in series with a $240V$, $60Hz$ power supply.

(i) What is the impedance of the circuit?

[4 marks]

(ii) What is the amplitude of the alternating current (peak current)?

[4 marks]

(iii) Calculate the phase angle between the current and the applied voltage. Does the current or the voltage reach its peak earlier?

[5 marks]

(b) In a woofer only low frequencies of the input signal of are audible (low pass filter). In a tweeter only high frequencies of the input signal are audible (high pass filter). Which speaker corresponds to a "woofer" and to a "tweeter", in Fig. 1? Show your analysis. *Tip: You may replace each speaker with a resistor, with a resistance R, and compare the voltage drop V_o across that resistor to the input voltage V_i at different frequencies of the input signal.*

[12 marks]

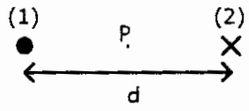

FIG. 2:

Question 3

(a) Write down an expression for Ampere's law in an integral form, and explain carefully the meaning of each term.

[2 marks]

(b) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both *inside* and *outside* the wire if the current density J is constant.

[12 marks]

(c) Two long parallel wires carry a current I and they are separated by a distance d. In wire (1) the current is out of the the page and in wire (2) the current is into the page as shown in the Fig. 2.

(i) Find the magnitude and the direction of the magnetic field at the midpoint P.

[4 marks]

(ii) Calculate the magnetic force per unit length felt by wire (2) due to the magnetic field that is produced by wire (1).

[7 marks]

FIG. 3:

Question 4

(a) The complex impedance in a AC-circuit containing resistors, capacitors and inductors is usually written as $\widetilde{Z} = R + iX$, where $i = \sqrt{-1}$.

(i) What does R and X represent?

[2 marks]

(ii) If the AC-circuit contains a resistor, capacitor and inductor all connected in series, what is the value of R, and X. Also calculate effective impedance $Z = |\widetilde{Z}|$ for this circuit.

[4 marks]

(iii) If the AC-circuit contains a resistor, capacitor and inductor all connected in parallel, what is the value of effective impedance $Z = |\widetilde{Z}|$ for this circuit.

[5 marks]

(b) An AC voltage source is connected to a "black box" as shown in Fig. 3. The circuit elements in the black box and their arrangement, are unknown. Measurements outside the "black box" provide the following information: $V(t) = (240V)\sin(\omega t)$, and $I(t) = (1.6A)\sin(\omega t + 30°)$.

(i) What is the phase angle in the circuit?

[2 marks]

(ii) Does the current leads or lags the voltage?

[2 marks]

(iii) Is the circuit in the "black box" largely capacitative or inductive?

[2 marks]

(iv) Is the circuit in the "black box" at resonance?

[2 marks]

(v) What is the power factor?

[2 marks]

(vi) Does the box contain a resistor? An inductor? A capacitor?

[2 marks]

(vii) Compute the average power delivered to the black box by the AC source.

[2 marks]

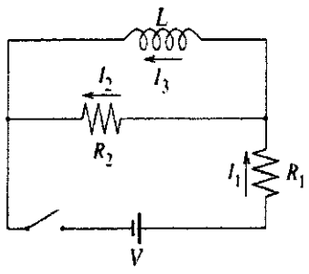

FIG. 4:

Question 5

(a) What analogies can you draw between an ideal solenoid and a parallel plate capacitor?

[4 marks]

(b) LC oscillators have been used in circuits connected to load speakers to create some sound of " electronic music". What inductance must be used with a $7.6\mu F$ capacitance to produce a frequency of 10 kHz, near the upper end of the audible range of frequencies.

[5 marks]

(c) The switch in Fig. 4 has been open for a long time. Determine the currents I_1,I_2, and I_3 in the resistors and in the self-inductor at the moment

(i) the switch is closed,

[4 marks]

(ii) a long time after the switch is closed.

[4 marks]

The internal resistance of the battery is negligible. Express your answer only in terms of V, R_1,R_2, and L.

(d) Replace the inductor in the above circuit with a capacitor C.Determine the currents I_1, I_2, and I_3 in terms of V, R_1,R_2, and C under the same conditions

[8 marks]

Question 6

(a) What is the principle of superposition?

[3 marks]

(b) A positive charge, q, is moved through a potential difference of δV. What is the amount of work that must be expended in order to achieve this change?

[5 marks]

(c) The length of the path taken by the charge between the points with potential difference of δV is doubled. By how much has the amount of work expended in moving the charge changed?

[2 marks]

(d) The magnitude of an electric field that lies in the xy plane of the figure can be represented by the equation

$$\mathbf{E} = k(y\hat{x} + x\hat{y})$$

where $\hat{x}$ and $\hat{y}$ are unit vectors in the x and y directions respectively.

(i). What are the units of k?

[3 marks]

(ii). Find the increase in potential energy of a positive charge of magnitude $3.20 \times 10^{-19}C$ when it is moved from a point with co-ordinates $(x = 3.0m, y = 4.0m)$, to the origin of the co-ordinate system, i.e. to the point $(x = 0, y = 0)$.

[12 marks]