UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2010

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) (i) Draw a unit cell of an body centered cubic (bcc) lattice. (2 marks)

(ii) Distinguish between a primitive unit cell and a conventional unit cell. (2 marks)

(iii) Draw the Wigner - Seitz cell of a two-dimensional direct lattice. State whether it is a primitive or a conventional cell. (2+1 marks)

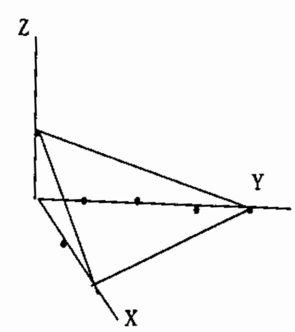
(b) (i) In the diagram of a cubic unit cell, show a (200) and a $(\bar{1}00)$ plane. (4 marks)

(ii) What is meant by packing fraction of a crystal? Determine the packing fraction of a bcc crystal (2+3 marks)

(c) (i) Write down the translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ of the primitive cell of an fcc lattice in terms of its lattice constant 'a'. (3 marks)

(ii) Use the above results to find the Miller indices of a (100) plane as referred to its primitive axes. (4 marks

(iii) Find the Miller indices of the plane in the figure below. (2 marks)



Question Two.

(a) (i) State ***Bragg's law*** in crystal diffraction. What is its physical meaning?

(2+2 marks)

(ii) Can visible light be used for diffraction experiments from a crystal? Explain.

(2 marks)

(b) Both x-rays and electrons can be used for crystal diffraction experiments. If the lattice constant of crystal is 1 Å, calculate:

(i) The energy- of x-ray photons in eV that can be used in diffraction experiment.

(ii) The energy of an electron that can be diffracted by the crystal.

[Given: photon energy $E = hc/\lambda$, Electron energy $E = h^2/(2m\lambda^2)$]

(4 + 4 marks)

(iii) State why electron beam cannot be used for study of bulk materials as compared to x-rays.

(2 marks)

(c) Show that the condition for x-ray diffraction from a plane in a bcc lattice is that the sum of the Miller indices of the plane should be an even number.

Given: The geometric structure factor of a crystal is:

$S_G = \sum_{j=1}^{s} f_j \exp[-i2\pi(n_1 h + n_2 k + n_3 l)]$, where 's' is the number of atoms in the basis and n_1, n_2, n_3 are fractional coordinates. 'f' is the atomic form factor.

(Assume all toms have the same atomic form factor)

(5 marks)

(d) According to the results in (c) above, there should be no diffraction lines corresponding to reflections from (100) planes in a bcc lattice. With the help of a diagram give a physical explanation of this phenomenon. (4 marks)

Question Three.

(a) Explain the phenomenon of photoconductivity in semiconductors. (4 marks)

(b) A p-type germanium sample has a resistivity of 40 Ω- cm at 300 K. It is illuminated with light that generates 10^7 excess holes. Calculate the change in conductivity of the sample caused by the light.

[Given: electron / hole mobilities of germanium are $\mu_n = 3900\ cm^{-2}\ V^{-1}s^{-1}$.
$\mu_p = 1900\ cm^{-2}\ V^{-1}s^{-1}$]

(5 marks)

(c) (i) With the help of a plot, explain how the electrical conductivity of an extrinsic semiconductor varies with temperature. (8 marks)

(ii) An intrinsic semiconductor has a resistance of 10 Ω at 364 K and 100 Ω at 333 K If this change is caused entirely by temperature variation, calculate the band gap of the semiconductor.

(8 marks)

Question Four.

(a) (i) Define Fermi Energy. (2 marks)

(ii) Write down the Fermi - Dirac (F- D) distribution function for a system of fermions. (2 marks)

(iii) Compute the values of the F-D distribution function for the following cases at absolute zero temperature.

1. energy of the fermion $\epsilon >$ Fermi energy ϵ_F
2. energy of the fermion $\epsilon <$ Fermi energy ϵ_F (4 marks)

(iv) In a single sketch show how the Fermi function varies with energy at T = 0K and also at T > 0K and comment on the physical meaning of your observations. (5 marks)

(b) (i) Using the Schrödinger wave equation, show that the energy of a free electron is:

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}$$

, where symbols have their usual, meanings. (4 marks)

(ii) Use the results in (i) above to show how the Fermi energy is related to the electron concentration, and hence derive an expression for the density of states of the electrons in a metal. (8 marks)

Question Five.

(a) Explain how electrical conductivity of a pure semiconductor can be increased by:

(i) Thermal generation of carriers
(ii) Doping.

Give examples where necessary (6 marks)

(b) With the help of appropriate energy band diagram, show that the density of electrons in the conduction band of a semiconductor is give by the expression:

$$2\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp\left[\frac{\varepsilon_F - \varepsilon_g}{kT}\right]$$

where symbols have their usual meanings. [Assume $(\epsilon - \epsilon_{F)} \ggg kT$]

Given: $\int_0^{\infty} \exp(-nx)x^{1/2}dx = \frac{1}{2n}\sqrt{\pi}$

(12 marks)

(c) A silicon sample is doped with 10^{17}cm^{-3}arsenic atoms. All dopants are ionised.

1. What is the equilibrium hole concentration?
2. Where is the Fermi level relative to the centre of the band gap?

[intrinsic carrier concentration of silicon is $1.5\text{x}10^{10}$ cm^{-3}]

(7 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$He atom		6.65×10^{-27} kg
Mass of $_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 l mol^{-1}