UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS.

MAIN EXAMINATION 2009/2010

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

THIS PAPER CONTAINS FIVE QUESTIONS

QUESTION ONE

(a) (i) Explain what is meant by **phase space**. (3 marks)

(ii) The volume of a state in phase space is said to be h^3 where **h** is Planck's constant. Verify dimensionally how far this statement is true.

(3 marks)

(b) (i) Define ***density of states*** in phase space. (2 marks)

(ii) Derive an expression for the volume element in phase space in terms of energy and show that density of states

$$g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon,$$

where the symbols have their usual meanings. (12 marks)

(c) Calculate the density of states at an energy level of $3.3\text{x}10^{-19}$ J in a system of electrons contained in a volume 10^{-4} m^3.

(5 marks)

QUESTION TWO.

(a) Show that for a classical system in thermal equilibrium, the Maxwell Boltzmann distribution function is $n_s = g_s \exp(\alpha + \beta\varepsilon_s)$, where the symbols have their usual meanings.

(11 marks)

(b) Compute an expression for the average energy of the most probable configuration of a classical system having two non-degenerate energy levels of energies ϵ and 2ϵ.

(5 marks)

(c) In the above example if $\epsilon = 1$ J and the total number of particles is 15000,

(i) find the values of α and β of the distribution function for the system with a total energy of 20,000 J.

(5 marks)

(ii) calculate the population of the two energy levels. (4 marks)

QUESTION THREE.

(a) Show that the entropy of a classical perfect gas $S = NkT \ln Z + \frac{E}{T}$ where Z is the partition function. (8 marks)

(b) Two equal volumes of the same gas each having entropy S, and at the same temperature and pressure, are mixed together.

(i) Compute the entropy of the mixture in terms of S using its expression in (a) above. Do you see any anomaly in your result? Explain. (6 marks)

(ii) On the assumption that the molecules of a classical gas are indistinguishable, the expression for entropy in (a) above can be modified as

$$S = \frac{E}{T} + Nk \ln \frac{Z}{N} + Nk$$

Verify whether or not this equation can resolve the anomaly.

Given: $Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$; $E = (3/2)\, kT$

Hint: Substitute for Z and E in the above expression. (4 marks)

(c) Calculate the entropy of one mole of helium gas (^4He) at 300 K from the following data:

Molar volume of the gas	=	$22.4 \times 10^{-3}\ m^3\ mol^{-1}$
Avogadro's number N_A	=	$6.02 \times 10^{23}\ mol^{-1}$
Planck's constant, h	=	6.63×10^{-34} J.s
Mass of He molecule	=	6.65×10^{-27} kg

(7 marks)

QUESTION FOUR.

(a) The quantum statistical expression derived by Max Planck for the spectral distribution of energy from a black body is expressed as:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

Derive the spectral distribution under short and long wavelength limits.

(8 marks)

(b) (i) Use the Planck's distribution function in (a) above to show that the total energy radiated is proportional to the fourth power of the absolute temperature of the body.
(See appendix1 for definite integrals) (10 marks)

(ii) Given that the proportionality constant in the expression obtained in (b) above for total energy is equal to σ (4/c), where σ is the Stefan-Boltzmann constant, and **c** is the velocity of light, calculate the value of σ.

(7 marks)

QUESTION FIVE.

(a) (i) Given that the density of states for a system of fermions:

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$, show that the Fermi energy of the system is:

$$\varepsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$, where the symbols have their usual meanings.

(8 marks)

(ii) Use the expression in a (i) for the Fermi energy to show that the average kinetic energy of a particle in a Fermi gas at 0 K is (3/5) times the Fermi energy.

[Hint: average energy = $(1/N)\int \epsilon \, n(\epsilon)d\epsilon$] (6 marks)

(b)

(i) Define the Fermi temperature T_F (2 marks)

(ii) Calculate T_F at 300 K for a metal with Fermi energy 3.12 eV.

(3 marks)

(iii) The electronic contribution to specific heat capacity is given as $C_V = 3Nk\,T / T_F$. Comment on the effect of T_F as calculated in (ii) above, on the specific heat .

(2 marks)

(c) Calculate the Fermi energy of a metal (in electron volts) having an electron density of $5\text{x}10^{28}$ m^{-3}. (4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\, x\, dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\, x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\, x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\, x^5\, dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\, e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\, dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Stefan - Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} Wm^{-2}K^{-4}
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}