UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2010/11

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) A projectile is launched with a velocity υ_0 at an angle θ with the horizontal in an incline at an angle ϕ where there is a constant gravitational acceleration g vertically downward. See Figure 1.

(i) Find the distance d the body travels up the incline in terms of g, θ and υ_0. **(10 marks)**

(ii) How would you determine the angle θ that would give the maximum distance reached up the incline. **(2 marks)**

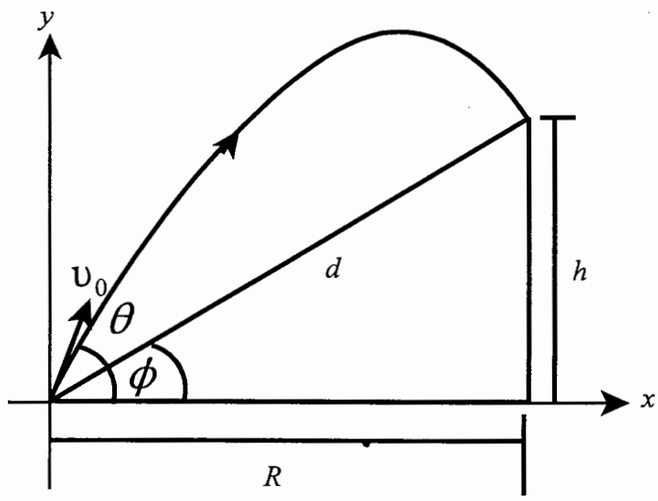

Figure 1.

(b) Plane polar coordinates

(i) Use a clear diagram as an aid to determine the polar unit vectors $\hat{r}$ and $\hat{\theta}$ in terms of Cartesian unit vectors $\hat{i}$ and $\hat{j}$. **(6 marks)**

(ii) Write down the acceleration in plane polar coordinates and give the meaning of each term. **(7 marks)**

QUESTION 2

(a) A body of mass m falls from rest under constant gravitational acceleration g through a viscous medium which resists the motion of the body by the viscous force $f = b\upsilon$ where b is a constant with appropriate units, and υ is the instantaneous velocity of the mass.

(i) Find the velocity of the body as a function of time. **(6 marks)**

(ii) Determine terminal velocity reached by the body. **(2 marks)**

(iii) Find the position of the body as a function of time. **(5 marks)**

(iv) Use the result in (iii), to determine how the position behaves as function of time at large times (t >> 0 s).. **(2 marks)**

(b) A single bead can slide without friction on a wire that is bent into a circular loop of radius R as in Figure 2. The loop is always in a vertical plane and rotates steadily about its vertical diameter such that the particle moves in a horizontal circle with velocity υ. The position of the bead is described by an angle θ that the radial line from the centre of the loop to the bead makes with the vertical. Determine the angle θ in terms of R, g and υ. **(10 marks)**

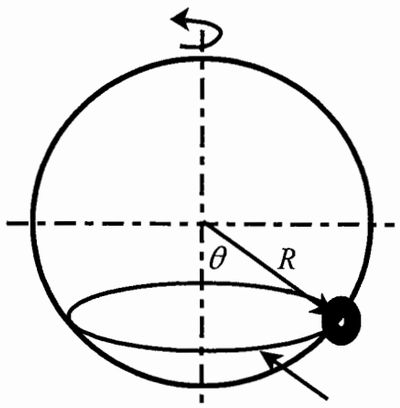

Figure 2. motion of bead in a horizontal circle

QUESTION 3

(a) A thin uniform rod of length L and total mass M is bent into an arc of a circle subtended by an angle of 90° (from $\theta_0 = 45°$ to $\theta_f = 135°$) where L makes a quarter of the circumference as shown in Figure 3.

(i) Find the element of mass dm in terms of the density M and $d\theta$.. **(5 marks)**

(ii) Find the centre of mass $\vec{R}$ of the bent rod in terms of L, taking the origin to be at the centre of curvature. **(6 marks)**

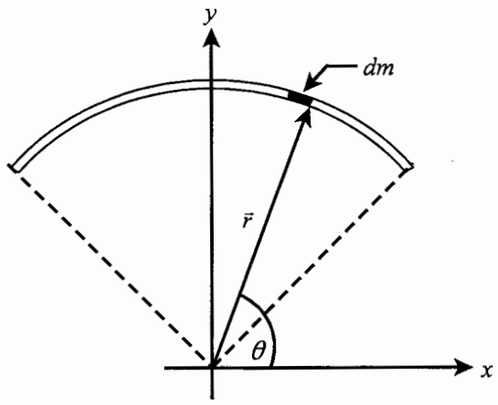

Figure 3.

(b) A rocket ascends from rest in a uniform gravitational field of magnitude g by ejecting exhaust gases with constant speed u with respect to the rocket. The rate at which exhaust gases are expelled is given $dM/dt = -\gamma M$, where M is the instantaneous mass of the rocket and γ is a constant with appropriate units, and that the rocket is retarded by air resistance with a force $f = Mb\upsilon$, where υ is the instantaneous velocity of the rocket and b a constant with appropriate units.

(i) Make sketches of the rocket at a time t and at time $t + \Delta t$. Between time t and $t + \Delta t$, the fuel burnt and ejected as exhaust is Δm. Indicate the velocity of the mass Δm and the mass M. **(2 marks)**

(ii) Develop the momentum equations at time t and at time $t + \Delta t$. **(2 marks)**

(iii) Develop the equation of motion for the rocket. **(5 marks)**

(iv) Determine the velocity of the rocket as a function of time. **(5 marks)**

QUESTION 4

(a) A mass at rest breaks into two pieces with masses m_1 and m_2 due to an internal activity. Between the small mass and larger mass, which ends up with higher kinetic energy? **(7 marks)**

(b) A block of mass m slides along a horizontal table with speed v_0. At $x_0 = 0$ it hits a horizontal spring of spring constant k and at the same time begins to experience a frictional force from the surface (see Figure 4). The coefficient of kinetic friction varies with x and is given by $\mu = bx$ where b is a constant. Use the work -energy theorem to find the velocity of the mass as a function of x on its first trip to the right on the surface with friction. **(8 marks)**

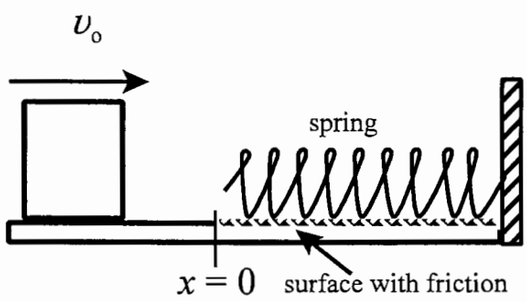

Figure 4.

(c) A conservative force acting on a particle along the x-axis varies as $F = -Ax + Bx^2$, where A and B are positive constants.

(i) Find the potential energy function associated with the force. **(4 marks)**
(ii) Determine the equilibrium points for the potential. **(2 marks)**
(iii) Determine the stability of the equilibrium points. **(2 marks)**
(iv) Find the frequency of small oscillations about the equilibrium point. **(2 marks)**

QUESTION 5

(a) Derivation the Parallel Axis Theorem for the moment of inertia. **(7 marks)**

(b) A uniform density thin plate of mass M in the form of a right angled triangle is placed on the xy plane with one end of the hypotenuse at the origin as shown in Figure 5. The plate has a base length L and height h, and is to be rotated about the y-axis located a distance $2L/3$ from the vertex with angle θ.

(i) Find the mass element dm of a very thin strip of area dA as shown in the Figure 5, in terms of L, x, M and dx. **(5 marks)**

(ii) Calculate the moment of inertia of the plate about the y-axis. **(7 marks)**

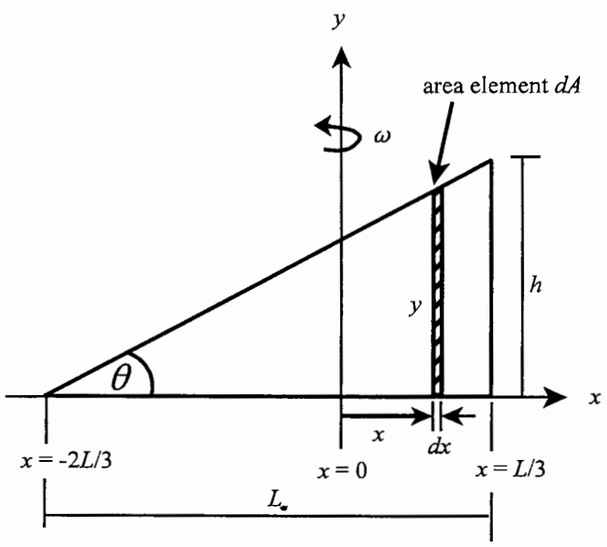

Figure 5.

(c) A particle of mass m moves in a circle of radius R at constant angular velocity ω (see Figure 6). Find the angular momentum for the particle about an axis through point P. **(6 marks)**

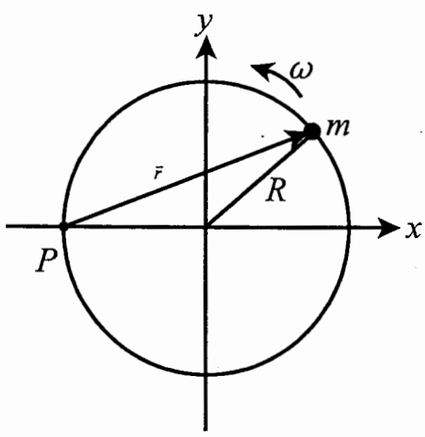

Figure 6.