UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMANTARY EXAMINATION: 2010/2011

TITLE OF THE PAPER: CLASSICAL MECHANICS

COURSE NUMBER: P320

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 MARKS.
- MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Question 1

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(a) What is a normal mode solution?

[3 marks]

(b) Two harmonic oscillators (with masses m and spring constants κ) are coupled by spring such that the Lagrangian of the system is given by

$$L = \frac{m}{2}(\dot{x_1}^2 + \dot{x_2}^2) - \kappa(x_1^2 + x_2^2) - \kappa_{12}(x_1 - x_2)^2$$

where x_1, x_2 are the displacements of the masses of the oscillators with respect to their equilibrium position and κ_{12} is the coupling spring constant.

(i) Obtain the frequencies of the normal modes of the system.

[10 marks]

(ii) Write down the normal mode solutions of the system. State which solution corresponds to a symmetric or asymmetric motion.

[12 marks]

Question 2

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(a) Consider three particles of equal mass m. The force $\mathbf{F_1} = 0.2f\,\hat{x} - 0.75f\,\hat{y}$ acts on particle (1), $\mathbf{F_2} = 0.8f\,\hat{x} + 0.75f\,\hat{y}$ acts on particle (2) and the force $\mathbf{F_3} = 2f\,\hat{x}$ acts on particle (3). If the particles are initially at rest at the origin, what is the position, velocity and the acceleration of their center of mass?

[9 marks]

(b) The center of gravity of a system of particles is the point about which external gravitational forces exert no torque. For a uniform gravitational force, show that the center of gravity is identical to the center of mass for the system of particles.

[7 marks]

(c) State **four** fictitious forces that can be felt by an observer in an accelerating frame. For each case write the expression of the force and give a physical example where the force is effective.

[9 marks]

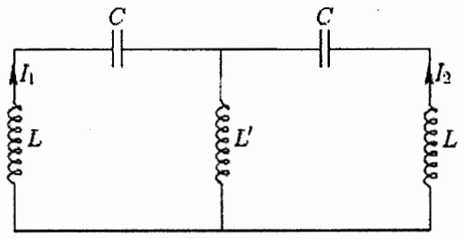

FIG. 1:

Question 3

(a) Draw and state the correspond Kirchhoff's equation for an electrical circuit that is equivalent to a

(i) simple harmonic oscillator

(ii) damped simple harmonic oscillator

(iii) forced damped harmonic oscillator

[9 marks]

(b) Find the normal frequencies and the normal modes of the coupled LC circuit shown in Fig. 1 in terms of $\omega_0 = 1/\sqrt{LC}$ and $\alpha = L'/L$.

[16 marks]

Question 4

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(a) Consider a rigid system consisting of four particles with equal mass m. The particles are located at the corner of a square with edge length $2a$. The center of mass for the system is a located at $(0, 0, z_0)$ and the system is rotating with a angular velocity
$\vec{\omega} = (0, 0, \omega_3)$.

(i) Calculate the inertial tensor $\mathbf{I}$ of the system.

[9 marks]

(ii) Calculate the angular momentum of the system.

[4 marks]

(iii) Calculate the kinetic energy of the system.

[4 marks]

(iv) Consider a point mass m traveling in a circle centered at the origin, with a radius r in the x-y plane, with an angular frequency $\vec{\omega} = (0, 0, \omega_3)$.

(i) Calculate the angular momentum of the particle.

[4 marks]

(ii) Let now the point center of circulation be $(0, 0, z_0)$. What is the angular momentum of the particle in this case?

[4 marks]

Question 5

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(a) Define a central force? Support your discussion by giving an example.

[4 marks]

(b) A particle moves in a potential $U(r) = -k/(3r^3)$, where k is a proportionality constant. The conservation of mechanical energy, E, means that

$$\frac{1}{2}m\dot{r}^2 + \frac{l_z^2}{2mr^2} + U(r) = E$$

where m and l_z are the mass and angular momentum of the particle, respectively.

(i) Sketch the effective potential felt by the particle.

[3 marks]

(ii) Find the radius of the stable circular orbit.

[6 marks]

(c) Consider a projectile launched from the surface of the Earth. Show that the escape velocity is

$$v_{escape} = \sqrt{2gR}$$

where R is the radius of the Earth (assumed to be spherical) and g is the acceleration on the surface of the Earth due to gravity.

[6 marks]

(d) State and derive Kepler's second law.

[6 marks]