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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2011/2012

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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Here are some integrals that may be useful in some questions.

$$\int_0^x (a^2 - x^2)x dx = -\frac{1}{3}(a^2 - x^2)^{3/2}$$

$$\int_0^x \frac{dx}{(a^2 - x^2)} = \frac{1}{2a} ln\left(\frac{a+x}{a-x}\right)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = arcsin\left(\frac{x}{a}\right)$$

QUESTION 1

(a) An ion at rest in the ionosphere is struck by a photon giving it an acceleration along the x-axis, that varies in time t as $a = a_0 \sin \omega t$, where a_0 and ω are constants.

(i) Find the velocity of the ion as a function of time. **(5 marks)**

(ii) What is the displacement x of the ion as a function of time? **(5 marks)**

(iii) Sketch the displacement-time graph for the particle, for $t = 0$ to $2T$, where T is the period. **(5 marks)**

(b) A particle moves in a circular path of radius r at constant angular velocity ω in the anticlockwise direction.

(i) Make a diagram of the trajectory of the particle and write down the position vector $\vec{r}$ in terms of r, ω and t using the Cartesian coordinate unit vectors. **(3 marks)**

(ii) Find the velocity of the particle as a function of time. **(2 marks)**

(iii) Find the acceleration of the particle. **(2 marks)**

(iv) Show that the velocity $\vec{v}$ is perpendicular to the position vector $\vec{r}$ at all times. **(3 marks)**

QUESTION 2

(a) An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Figure 1). The coefficient of static friction between the person and the wall is μ_s, and the radius of the cylinder is R, and gravity is downward.

(i) Find the maximum period T of the drum necessary to keep the person from falling, in terms of μ_s, R, and g. **(8 marks)**

(ii) Explain why there is a requirement for a maximum period. **(4 marks)**

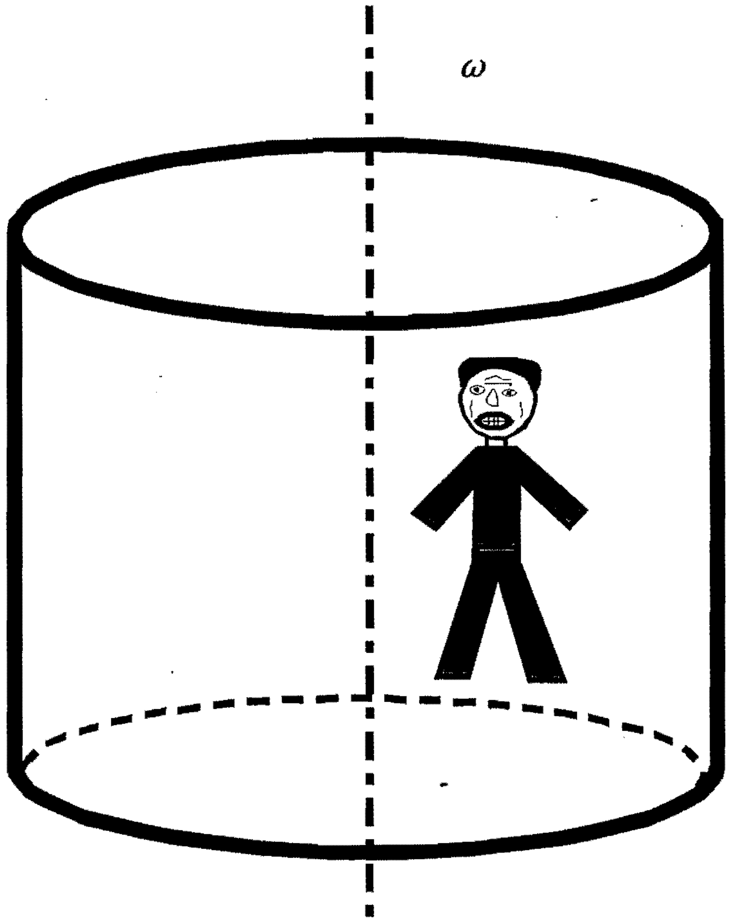

Figure 1.

(b) A synchronous satellite of mass m_s orbits the earth of mass M_E at a distance r from the centre of the earth.

(i) Find the tangential speed v of the satellite and in terms of G, M_E and its period T. **(8 marks)**

(ii) To appreciate the velocities and distances involved, find the tangential velocity in km/h and the radius of orbit of the satellite in km for synchronous satellite orbits that appear stationary with respect to the earth. Use the following data: $G = 6.67 \times 10^{-11}$ Nm^2/kg^2 $M_E = 5.98 \times 10^{24}$ kg. **(5 marks)**

QUESTION 3

(a) The object illustrated in Figure 2 is made of a thin flat uniform sheet of area density σ and mass M is in the form of a semicircle of radius a. Find the centre of mass of the object. The use of symmetry is allowed. **(9 marks)**

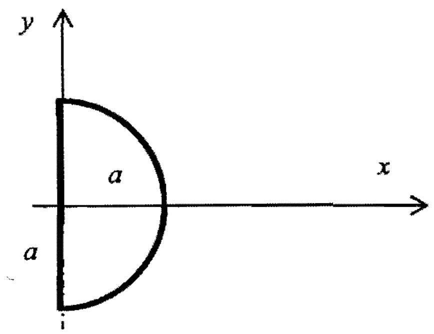

Figure 2.

(b) A raindrop of initial mass M_0 starts falling from rest under the influence of gravity. Assume that the drop gains mass from the cloud at a rate proportional to its instantaneous mass M and its instantaneous velocity v:

$$\frac{dM}{dt} = kMv,$$

where k is a constant. Neglect air resistance.

(i) Develop the equation in the form of an integral that can enable you to determine the velocity of the raindrop as a function of time as it moves through the cloud. **(8 marks)**

(ii) Find the expression for the velocity and show that eventually it becomes effectively constant. **(8 marks)**

QUESTION 4

(a) Derive an expression for the work-energy theorem in one dimension. **(9 marks)**

(b) Show that a body acted upon by the force $F = -kx$, oscillates harmonically when let go after undergoing some displacement x_0. **(10 marks)**

(c) Show that a body with the following forms of potential energy and kinetic energy oscillates harmonically. **(6 marks)**

$U = \frac{1}{2}Aq^2 + \text{constant}$

$K = \frac{1}{2}B\dot{q}^2,$

where q is a variable suitable to the problem, and A and B are constants.

QUESTION 5

(a) Find the acceleration of the Atwood's machine shown in Figure 3. The moment of inertia of the pulley is $I = (m_p R^2)/2$, where m_p is the mass of the pulley and R is its radius. **(12 marks)**

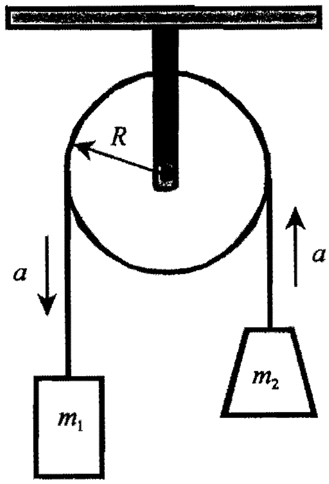

Figure 3.

(b) A drum of radius R rolls without slipping, down an inclined plane with friction as shown in Figure 4. The forces acting on the sphere are the weight mg, friction f and the normal force N, and are illustrated in the figure where they act.

(i) What is the torque on the drum about the centre of mass? **(5 marks)**

(ii) What is the torque on the drum about the origin A? **(8 marks)**

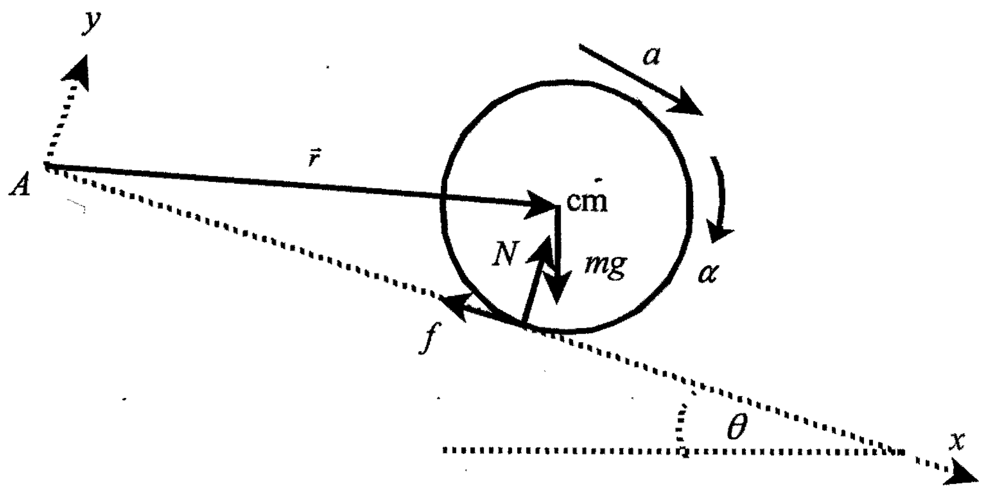

Figure 4.