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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2011/2012

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY **FOUR** OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) The velocity of a particle as a function of time is given by $v(t) = \frac{b}{k}(1 - e^{-kt})$, where b and k are constants with appropriate units and v is the instantaneous velocity.

i. Find the acceleration of the particle. **(4 marks)**

ii. Find the displacement y of the particle as a function of time? **(4 marks)**

iii. How does the displacement-time graph look like for long times (for $t >> 1$). **(4 marks)**

(b) Prove the sine law using the cross product. **(6 marks)**

(c) An elevator ascends from the ground with uniform velocity v_0. After a time T_1 a boy inside the elevator drops a marble through the floor. The marble is acted upon by the constant downward acceleration g due to gravity. It hits the ground a time T_2 later. Determine the height h of the elevator at time T_1 in terms of g, T_1 and T_2. **(7 marks)**

QUESTION 2

(a) The two blocks shown in Figure 1 are connected by a string of negligible mass. Find an expression for the horizontal displacement x of m_1, after a time t. Neglect friction and the mass of the pulley. **(9 marks)**

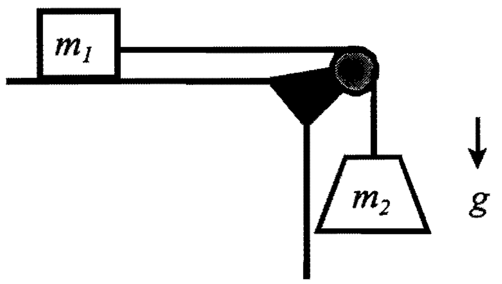

Figure 1.

(b) A smooth hemispherical bowl of radius R is fixed with its rim horizontal and uppermost. A smooth round particle of mass m moves without friction inside the bowl with a constant velocity υ in a horizontal circle of radius r. The particle makes an angle θ with the vertical as shown in Figure 2.

i. Make a resolved force diagram for the mass m. **(5 marks)**

ii. Determine an equation that relates the angle θ, with the velocity υ, the radius R, and the gravitational acceleration g. **(5 marks)**

iii. Find the angle θ if ——. **(6 marks)**

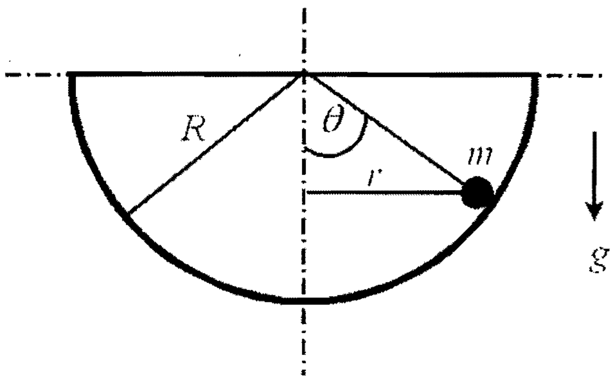

Figure 2.

QUESTION 3

(a) A rod of non uniform length-density $\lambda = \frac{\lambda_0 x}{L}$, where λ_0 is a constant, x is the distance from the lighter end and L the length of the rod, is placed along the x-axis as shown in Figure 3. Find the centre of mass $\vec{R}$ of the rod. **(7 marks)**

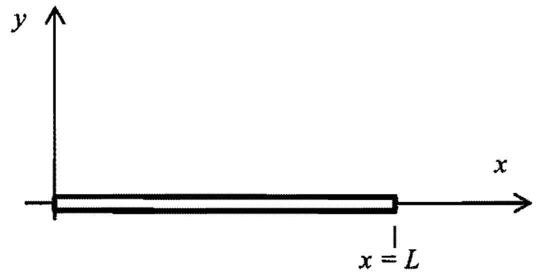

Figure 3.

(b) A sand-spraying locomotive sprays sand with a horizontal velocity $u + v$ into a freight car of mass M_0 as shown in Figure 4. The locomotive and freight car are not attached. The engineer in the locomotive maintains his speed so that the distance to the freight car is always constant. The sand is transferred at a constant rate dm/dt. Use the mass and momentum transport method to find an expression for the velocity of the freight car after it has accumulated sand of mass m, making the total mass of the freight car to be $M_f = (M_0 + m)$. **(12 marks)**

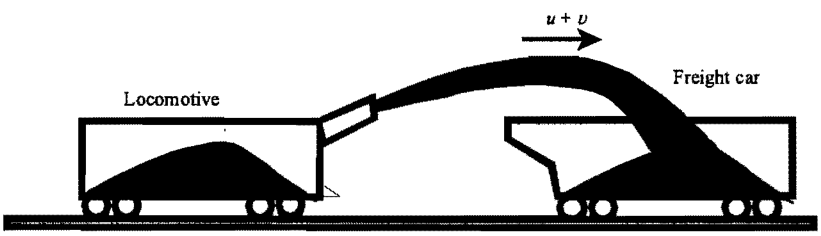

Figure 4.

(c) Water shoots out of a fire hydrant having nozzle diameter D with nozzle speed v_0, as illustrated in Figure 5. Find the reaction force on the hydrant in terms of the density of water ρ, the diameter of the nozzle D and the nozzle speed v_0. **(6 marks)**

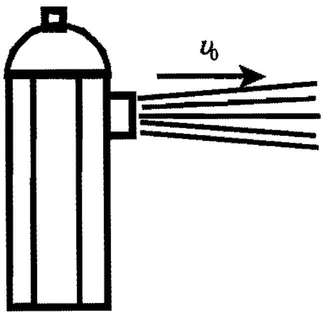

Figure 5.

QUESTION 4

(a) A body of mass m is projected upward with a velocity v_0 under a constant downward gravitational field of magnitude g. Use the work-energy theorem to determine the highest point reached by the body. **(4 marks)**

(b) A small block of mass m starts from rest at point A along a frictionless loop-the-loop as shown in Figure 6. Find the initial height h for the mass to just complete the circular part of the loop. Note: To just complete the loop the only force acting on the mass at the highest point B of the loop is the weight mg. **(8 marks)**

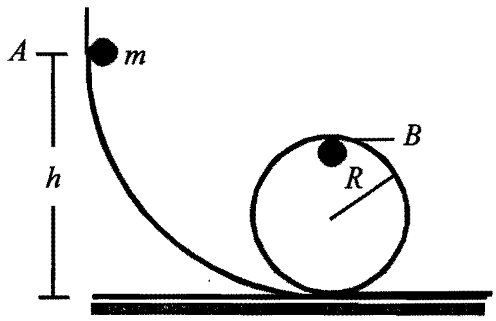

Figure 6.

(c) The potential energy function of a body moving along the x-axis is given by

$U = 3Ax^3 - 72Bx,$

where A and B are positive constants with appropriate units.

i. Find the force acting on the particle. **(3 marks)**

ii. Find the equilibrium points. **(4 marks)**

iii. Test the equilibrium points for stability. **(4 marks)**

iv. Find the angular frequency of small oscillations when the body is slightly disturbed from the stable equilibrium point. **(2 marks)**

QUESTION 5

(a) Find the moment of inertia of a uniform stick of mass M and length L about an axis through one of its ends. **(6 marks)**

(b) A body whose position vector is given by $\vec{r} = \left(at^3\hat{\imath} - bt^2\hat{\jmath} - ct\hat{k}\right)$ m is acted upon by a force $\vec{F} = \left(d\hat{\imath} + e\hat{\jmath} - f\hat{k}\right)$ N, where a, b, c, d, e and f are constants with appropriate units, and t is the time in seconds. Find the torque on the body. **(4 marks)**

(c) The angular momentum of a body is given by the equation:
$\vec{L} = \left(a\cos\omega t\,\hat{\imath} + a\sin\omega t\,\hat{\jmath} - bt\hat{k}\right)$, where a, and b, are constants with appropriate units, and t is the time in seconds. Find the torque on the body at any time t.
(3 marks)

(d) Show that a physical pendulum can oscillate harmonically when slightly disturbed from the pivot point. Also determine the angular frequency of its oscillation in terms of l, g and the radius of gyration k,. **(12 marks)**