UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE 41

DEPARTMENT OF PHYSICS

EXAMINATION: 2011/2012

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:
Answer any four Questions.
Each Question carries 25 Marks.
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Question 1

(a) Sketch the electric field lines around a spherical conductor exposed to a uniform electric field.

[2 marks]

(b) An isolated conductor of arbitrary shape has a net charge of $+7\mu$C. Inside the conductor is a cavity within which is a point charge $q = +3\mu$C. What is the charge

(i) on the cavity wall and

[3 marks]

(ii) on the outer surface of the conductor.

[3 marks]

(c) Consider a metallic spherical shall of radius a and charge Q.

(i) Find the electric potential everywhere.

[10 marks]

(ii) Calculate the potential energy of the system.

[5 marks]

(d) Suppose an electric field $\mathbf{E}(x, y, z) = (ax, 0, 0)$ where a is a constant. What is the charge density that corresponds to this field.

[2 marks]

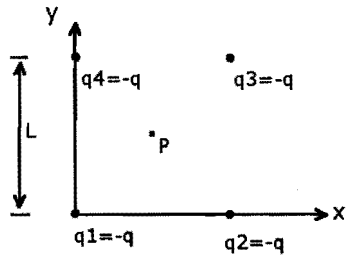

FIG. 1:

Question 2

(a) Four equal charges, $q1 = q2 = q3 = q4 = -q$, are situated at the four corners of a square of an edge length L, as shown in Fig. 1.

(i) Calculate the electric field at the center of the square, point P. Show the details of your calculations.

[6 marks]

(ii) Suppose the charge at the third corner is removed, what is the net electric field at point P?

[3 marks]

(iii) How much work is done (by a external agent) to bring back the charge $-q$ to the third corner from infinity?

[5 marks]

(iv) How much work is done to assemble the four charges in the configuration given by Fig. 1?

[4 marks]

(b) The electric potential in a certain region is given by $V(x, y) = a(x^2 + y^2) + bx + c$, where $a = 1V/m^2$, $b = -2V/m$ and $c = 5V$.

(i) Find the electric field at a point where $x = 1m$ and $y = 3m$.

[4 marks]

(ii) Determine the position where the electric field is zero.

[3 marks]

Question 3

(a) A charge q is released in a uniform magnetic field. According to Newton's second law the particle will experience an acceleration

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{v} \times \mathbf{B})$$

Assuming that the magnetic field is into the page, $\mathbf{B} = B_0(-\hat{\mathbf{z}})$. Then

$$m\frac{dv_x}{dt} = -qB_0v_y, \quad m\frac{dv_y}{dt} = qB_0v_x, \quad \text{and} \quad m\frac{dv_z}{dt} = 0,$$

where $\mathbf{v} = (v_x, v_y, v_z)$.

(i) Show that the particle will move in a circular path about the axis of the magnetic field if it is released an initial velocity $\mathbf{v}(t = 0) = v_0\hat{\mathbf{x}}$.

[8 marks]

(ii) What is the radius of the circular orbit?

[2 marks]

(iii) Sketch the trajectory of the particle when $\mathbf{v}(t = 0) = v_0(\hat{\mathbf{x}} + \hat{\mathbf{z}})$

[2 marks]

(b) A particle of charge q moving with a velocity $\mathbf{v} = v_0\hat{\mathbf{x}}$ enters a region of uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. The field deflects the particle towards the positive y-direction. Is the charge positive or negative.

[2 marks]

(c) Show that magnetic forces do no work.

[3 marks]

(d) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field both inside and outside the wire if the current is uniformly distributed over the outside surface of the wire.

[8 marks]

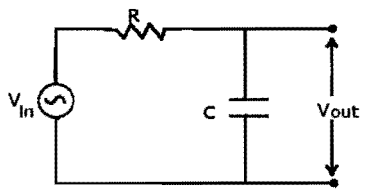

FIG. 2:

Question 4

(a) (i) If the frequency is halved in a series RLC circuit, what happens to the resistance, inductive reactance and capacitance reactance?

[6 marks]

(ii) What is the impedance of an RLC circuit at the resonance frequency?

[2 marks]

(b) The RC low-pass filter shown in Fig. 2 has a resistance $R = 90\Omega$ and capacitance $C = 8\mu F$. Calculate the gain V_{out}/V_{in} and the phase angle for an input frequency.

(i) $f = 600Hz$

[5 marks]

(ii) $f = 600kHz$

[5 marks]

(iii) Comment on the two results above

[2 marks]

(c) A RLC circuit consists of a 150Ω resistor, a $21\mu F$ capacitor and a $460mH$ inductor, connected in series with a $120V, 60Hz$ power supply. Calculate the phase angle between the current and the applied voltage. Does the current or the voltage reach its peak earlier?

[5 marks]

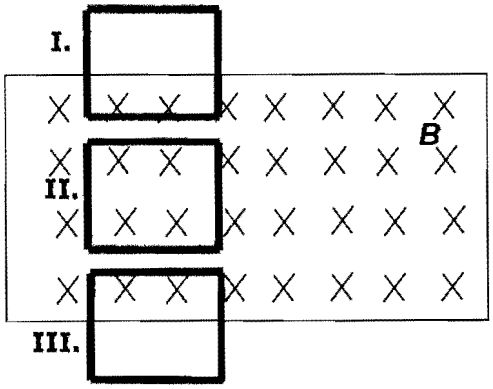

FIG. 3:

Question 5

(a) State Faraday's laws, describe every term in the expression.

[3 marks]

(b) State Lenz's law.

[2 marks]

(c) Figure 3 is a schematic diagram of a conducting loop falling through a uniform magnetic field. The field points into the page. The loop is located in the plane of the paper and is moving downwards under the influence of gravity. The diagram shows three positions of the loop: (I) While it is entering the magnetic field, (II) while it is moving inside the magnetic field, and (III) while it is exiting the magnetic field.

(i) For each step of the loop explain whether the magnitude of the magnetic flux through the loop Φ_B is increasing, constant or decreasing.

[6 marks]

(ii) For each step of the loop explain whether the induced current is clockwise, counter-clockwise or zero.

[6 marks]

(iii) How does the velocity of the loop change, while it is completely inside the magnetic field. Consider the forces acting on the loop in this case.

[8 marks]