UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2011/2012

TITLE OF THE PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:
Answer any four Questions.
Each Question carries 25 Marks.
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 6 PAGES, INCLUDING THIS PAGE.

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Question 1

(a) Write down Gauss's law in a differential form, and explain carefully the meaning of each symbol in the expression.

[2 marks]

(b) Suppose an electric field $\mathbf{E}(x, y, z) = (ax, be^{-cy}, 0)$ where a, b, and c are phenomelogical constants. What is the charge density that corresponds to this field.

[3 marks]

(c) If there are more electric field lines leaving a Gaussian surface than there are entering the surface, what can you conclude about the net charge enclosed by that surface.

[2 marks]

(d) If the charge inside a closed surface is known but the distribution of the charge is unspecified, can you use Gauss' law to find the electric field? Explain.

[3 marks]

(e) A thin spherical shell of radius a has a charge $+Q$ evenly distributed over its surface.

(i) Find the electric field $\mathbf{E}$ at a distance r from the center of the shell, where $r < a$, and $r > a$

[7 marks]

(ii) Sketch electric field E as a function of r.

[2 marks]

(iii) Do you think that the electric potential inside the spherical shell is zero or non-zero? Explain.

[3 marks]

(iv) Do you think that the electric potential difference between two points inside the spherical shell is zero or non-zero? Explain.

[3 marks]

Question 2

(a) Calculate the intensity of the magnetic field $\mathbf{B}$ inside and outside of a long straight cylindrical wire of radius a with a steady current I,

(i) if the current is uniformly distributed over the volume of the wire.

[9 marks]

(ii) if the current is uniformly distributed over the outside surface of the wire.

[9 marks]

(b) Suppose that the current through a conductor decreases exponentially with time according to

$$i(t) = I_0 e^{-t/\tau}$$

where I_0 is the initial current (at $t = 0$) and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor.

(i) How much charge passes this point between $t = 0$ and $t = \tau$?

[3 marks]

(ii) How much charge passes this point between $t = 0$ and $t = 10\tau$?

[2 marks]

(iii) How much charge passes this point between $t = 0$ and $t = \infty$?

[2 marks]

Question 3

(a) The complex impedance in a AC-circuit containing resistors, capacitors and inductors is usually denoted as $\widetilde{Z} = R + iX$, where $i = \sqrt{-1}$.

(i) What does R and X represent?

[2 marks]

(ii) If the AC-circuit contains a resistor, capacitor and inductor all connected in series, what is the value of R, and X. Also calculate effective impedance $Z = |\widetilde{Z}|$ for this circuit.

[4 marks]

(b) An AC voltage source $V(t)$ is connected to a "black box". The circuit elements in the black box and their arrangement, are unknown. Measurements outside the "black box" show that the voltage across it is $V(t) = (240V)\sin(\omega t)$ and the current pass through it is $I(t) = (2A)\sin(\omega t + \pi/6)$.

(i) What is the phase angle in the circuit?

[2 marks]

(ii) Does the current leads or lags the voltage?

[2 marks]

(iii) Is the circuit in the "black box" largely capacitative or inductive? Explain.

[3 marks]

(iv) Is the circuit in the "black box" at resonance? Explain.

[3 marks]

(v) What is the power factor?

[3 marks]

(vi) Does the box contain a resistor? An inductor? A capacitor?

[3 marks]

(vii) Compute the average power delivered to the black box by the AC source.

[3 marks]

Question 4

(a) If a current is passed through a coiled spring, does the spring stretch or compress? Explain.

[4 marks]

(b) Can a charge particle move through a uniform magnetic field without experiencing any force? Explain.

[3 marks]

(c) What type of magnetic field can exert a force in a magnetic dipole?

[3 marks]

(d) Show that magnetic forces do no work.

[4 marks]

(e) A circular loop of wire with a radius d is placed in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field. The magnetic filed varies with time according to $B(t) = B_0 + B_1 \sin(\lambda t)$, where B_0 and B_1 are positive constants.

(i) Calculate the magnetic flux Φ_B through the loop at time $t = 0$.

[5 marks]

(ii) Using Faraday's law calculate the induced electromotive force in the loop at $t = 0$.

[5 marks]

(iii) Consider that the resistance of the wire is R, what is the magnitude of the induced current at $t = 0$?

[1 marks]

Question 5

(a) Consider an electric dipole (*a configuration two point charges* q_1 *and* q_2 *with* $q_1 = -q_2 = q$, *at a distance* d *apart*).

(i) Calculate the electric field (magnitude and direction) at a distance y above the midpoint of the electric dipole

[9 marks]

(ii) What is the electric field due the electric dipole when $y >> d$.

[2 marks]

(b) Two very large planes are placed parallel to each other. They are uniformly charged with equal but opposite surface charge density $\pm\sigma$. Find the electric field in each of the three regions:

(i) to the left of both,

[4 marks]

(ii) between them,

[4 marks]

(iii) and to the right of both.

[4 marks]

(iv) If one releases an electron between the two plates, what is the direction of Coulomb force on it?

[2 marks]