UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2011/12

TITLE O F PAPER: MODERN PHYSICS & WAVE OPTICS

COURSE NUMBER: P231

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SEVEN PAGES INCLUDING THE COVER PAGE

THE LAST PAGE CONTAINS INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GRANTED BY THE CHIEF INVIGILATOR

QUESTION 1

a) Generally what conditions should be met by the path difference for two beams to interfere constructively and to interfere destructively? **(6 marks)**

b) Two waves each of wavelength $\lambda = 500$ nm have a phase difference $\phi = 13.2$ rad. Find the path difference between the two waves. **(3 marks)**

c) A P231 student performs a Young's double slit experiment using slits of unknown separation. He places the screen 2 m from the slits, and uses the 589.2 nm light from a sodium lamp. He observes that the tenth dark fringe occurs at 7.26 mm from the centre of the central maximum. Determine the spacing of the slits d. **(6 marks)**

d) The movable mirror in a Michelson interferometer is displaced by a distance Δl, and during this displacement 250 fringe reversals are counted. The light used is from a HeNe laser at 632.8 nm. Find the mirror displacement Δl. **(4 marks)**

e) The two path lengths in a Michelson interferometer are made equal, and an evacuated cylinder of length $L = 10$ cm having glass plates on each side is introduced in one arm. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. The number of bright fringes that pass through the screen is $N = 250$ and the laser light used is the 632.8 nm light from a HeNe laser. Determine the refractive index of the gas. **(6 marks)**

QUESTION 2

a) Light of wavelength $\lambda = 587.5$ nm illuminates a single slit 0.250 mm in width and the interference pattern is observed on a screen 2.00 m from the slit.

   i. Find the distance between successive minima. **(6 marks)**

   ii. If the slit is then replaced with another of width 12 mm, find the distance between successive minima, and comment on the image observed on the screen compared to the geometry of the slit. **(4 marks)**

b) The hydrogen spectrum has a red line at 656 nm and a blue line at 434 nm. What are the angular separations between the two spectral lines obtained with a diffraction grating that has 4500 grooves/cm in the first order? **(5 marks)**

c) Discuss, with the aid of diagrams and stressing the important points, the polarization of light by double refraction (birefringence) **(10 marks)**

QUESTION 3

a) Discuss with the aid of diagrams and equations the photoelectric effect, and state with justifications whether it supports the wave or particle nature of light. **(12 marks)**

b) X-rays having energy of 300 keV, undergo Compton scattering from a target. The scattered rays are detected at 37° relative to the direction of the incident rays.

   i. Determine the Compton shift at this angle. **(3 marks)**

   ii. Find the energy of the scattered X-ray. **(6 marks)**

c) A proton is accelerated through a potential of 10 000 V. Find the de Broglie wavelength for this proton. **(4 marks)**

QUESTION 4

a) Consider a hydrogen-like beryllium ion (Be^{3+}), Z = 4.

   i. Use the procedure developed by Bohr for the hydrogen atom to find the expression for the energy levels of the ion, and calculate the energies for the first 4 states, and comment on the energy gaps between successive states. **(10 marks)**

   ii. Find the ionization energy of the ion. **(3 marks)**

b) A particle in an infinite potential well from $x = 0$ to $x = L$ in the $n = 1$ state has the wave function: $\psi(x) = \sqrt{\frac{2}{L}} sin\left(\frac{\pi x}{L}\right)$. Find an expression for the probability of finding the particle anywhere from $x = 0$ to $x = l$, where $0 < l < L$. **(6 marks)**

c) List the three quantum numbers for the quantum model of the hydrogen atom and give their ranges. **(3 marks)**

d) State the Pauli Exclusion Principle. **(3 marks)**

QUESTION 5

a) An alpha particle having an initial energy of 0.500 MeV is incident on a gold atom ($^{197}_{79}Au$).

   i. Use energy methods to calculate the distance of closest approach for a head-on collision between the alpha particle and the gold nucleus at rest. Assume the gold nucleus is so heavy that it remains at rest during the collision. **(5 marks)**

   ii. If the effect of the electrons was included how would it influence the answer obtained? **(2 marks)**

b) A star of mass 3.978 x 10^{30} kg (twice the size of the sun) ends its life by combining its protons and electrons to form a neutron star. Find the final radius of the star and compare this radius to its original radius of 876 275 km. **(4 marks)**

c) The nucleus $^{15}_{8}O$ decays by electron capture.

   i. Write the equation for the reaction in the nucleus. **(2 marks)**

   ii. Write the equation for the reaction in terms of neutral atoms. **(2 marks)**

   iii. Disregarding the recoil of the daughter element, determine the energy of the neutrino emitted. **(4 marks)**

d) It has been estimated that the available uranium on earth is in the order of 10^9 tons, of which 0.7% is the fissionable ^{235}U isotope that releases 208 MeV per fission reaction. Assume that the world energy use is 474 x 10^{18} J per year. If the energy consumption remained constant due to continuous improvements in efficiency, how long would the power supply last if all the world energy were to be optained from the fission of ^{235}U isotopes? **(6 marks)**

SOME INFORMATION THAT MAY BE USEFUL IN SOME PROBLEMS

Avogadro's number $A = 6.02 \times 10^{23}$ particles per mole
Coulomb constant $k_e = 8.987\ 551\ 788 \times 10^9$ N.m^2/C^2
Boltzmann's constant, $k_B = 1.3801 \times 10^{-23}$ J/K
Bohr radius $a_0 = 5.291\ 772 \times 10^{-11}$ m
Bohr magneton, $\mu_B = 9.27 \times 10^{-24}$ J/T
Planck's constant, $h = 6.626\ 075 \times 10^{-34}$ Js
$\hbar = 1.054\ 572 \times 10^{-34}$ Js
$hc = 1.986\ 447 \times 10^{-25}$ Jm
$2\pi hc^2 = 3.741\ 859 \times 10^{-15}$ J·m^2·s^{-1}
Radii of orbit for the hydrogen atom $r_n = n^2 a_0$
Rydberg constant $R_H = 1.097\ 373 \times 10^7$ m^{-1}.
Speed of light in vacuum, $c = 2.997\ 924\ 58 \times 10^8$ m/s
Stefan-Boltzmann Constant $\sigma = 5.669\ 6 \times 10^{-8}$ W/(m^{2}K^2)
Wien's displacement law $\lambda_{max} = \frac{hc}{4.965 k_B T}$

Nuclear Data

Electron charge, $e = 1.602\ 177\ 33 \times 10^{-19}$ C
Electron mass, $m_e = 9.109\ 389\ 7 \times 10^{-31}$ kg = 0.000 548 6 u
Neutron, $m_n = 1.674\ 928\ 6 \times 10^{-27}$ kg = 1.008 665 u
Proton mass, $m_p = 1.672\ 623 \times 10^{-27}$ kg = 1.007 276 u

1 atomic mass unit = 1 amu = 1 u = $1.660\ 540\ 2 \times 10^{-27}$ kg ≡ 931.494 MeV rest mass energy
1 eV = $1.602\ 177\ 33 \times 10^{-19}$ J: 1 MeV = $1.602\ 177\ 33 \times 10^{-13}$ J
$T_{1/2}(^{14}C)$ = 5730 years
Ratio of carbon 14 to carbon 12 in the atmosphere $= \frac{N(^{14}C)}{N(^{12}C)} = 1.2987 \times 10^{12}$
1Curie (Ci) = 3.7×10^{10} Becquerel (Bq)
$r_0 = 1.2 \times 10^{-15}$ m

Cerium (^{140}Ce) atomic mass = 139.905 434 u
Deuterium (^{2}D) atomic mass = 2.014 102 u
Helium (^{4}He) atomic mass = 4.002 603 u
Hydrogen (^{1}H) atomic mass = 1.007 825 u
Hydrogen molecular mass = 1.007 94 u
Iron (^{56}Fe) atomic mass = 55.934 942 u
Molybdenum (^{94}Mo) mass = 93.905 088 u
Neodymium (^{144}Nd) atomic mass = 143.910 083 u
Oxygen ^{15}O atomic mass = 15 .003 065 u
Nitrogen ^{15}N atomic mass = 15.000 109 u
Ruthenium (^{98}Ru) atomic mass = 97.905 287 u
Tritium (^{3}T) atomic mass = 3.016 049 u

$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$