UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

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MAIN EXAMINATION	:	2011/2012
TITLE OF PAPER	:	THERMODYNAMICS
COURSE NUMBER	:	P242
TIME ALLOWED	:	THREE HOURS
INSTRUCTIONS	:	ANSWER ANY FOUR OUT OF FIVE QUESTIONS
		EACH QUESTION CARRIES 25 MARKS
		MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

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INFORMATION

Universal gas constant, R	$= 8.314\ J\ mol^{-1}K^{-1}$
Specific heat of water, c_w	$= 4186\ J\ kg^{-1}K^{-1}$
Density of water, ρ	$= 10^3\ kg.m^{-3}$
Latent heat of fusion of ice, L_f	$= 3.33 \times 10^5\ Jkg^{-1}$
Latent heat of vaporisation of water, L_v	$= 2.256 \times 10^6\ Jkg^{-1}$
Specific heat of iron, c_i	$= 448\ J\ kg^{-1}K^{-1}$
Avogadro's number, N_A	$= 6.02 \times 10^{23}$ molecules.mol^{-1}
Boltzmann constant, k_B	$= 1.38 \times 10^{-23}\ JK^{-1}$
Stefan-Boltzmann constant, σ	$= 5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
1 atmosphere	$= 1.013 \times 10^5 Nm^{-2}$
Thermal conductivity of glass, k_{glass}	$= 0.8\ Wm^{-1}K^{-1}$
Thermal conductivity of air, k_{air}	$= 0.0234\ Wm^{-1}K^{-1}$

For a monatomic gas, $\gamma = 5/3$; $C_v = 3R/2$, where $R = 8.314\ Jmol^{-1}\ K^{-1}$.

For a diatomic gas, $\gamma = 7/5$

QUESTION 1

(a) Show that, for an ideal gas with constant heat capacities, the slope of an adiabatic curve is negative in a p-V diagram and that it has a larger absolute value than the slope of an isothermal curve at the same values of p and V. Assume that the process is quasistatic. (10 marks)

(b) The operation of a Carnot engine is represented by the cycle shown in Fig. 1.

(i) What thermodynamic processes are involved in steps SP and QR; PQ and RS? (2 marks)

(ii) Write an expression for the thermal efficiency of the Carnot engine. (2 marks)

(iii) If the above is an ideal steam engine with T_H = 450 K, operating at room temperature (300 K), calculate the efficiency of the engine. (2 marks)

(c) A commercial refrigerator is kept in a room at 27°C. In order to maintain the freezer compartment of the refrigerator at -30°C during the summer season, heat is removed from the freezer at the rate of 1.4 kJs^{-1}.

(i) Calculate the coefficient of performance of the freezer. (2 marks)

(ii) Determine the power that must be supplied to the freezer in order to maintain the freezer temperature at -30°C. (2 marks)

(d) Two kilogrammes of water, initially at 10°C, is brought in contact with a heater. Calculate the change in entropy of the water when its temperature has reached 100°C. (5 marks)

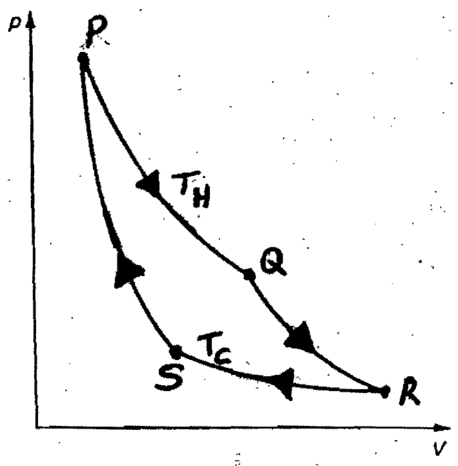

Fig. 1

QUESTION 2

(a) Explain how you would measure the thermal conductivity of a conductor. Use a suitable equation(s) and a schematic diagram to illustrate your point. (10 marks)

(b) Fig. 2 shows the cross section of a vessel in the form of a spherical shell. The inner and outer radii of the shell are r_1 and r_2, respectively. The shell wall has a coefficient of thermal conductivity, k. The inner sphere is maintained at a temperature T_1 and the outer sphere is at a temperature T_2.

Derive an expression for the rate of heat flow through the wall between the two concentric spheres. (7 marks)

(c) A room consists of a window, 8.0 cm^2 in area. The window is made of two layers of glass which are separated by an air space of 4.0 mm. The thickness of each of the glass layers is 6.0 mm.

How much heat would be lost through the window per second if the inside of the window is at 30°C whilst the outside is at -10°C? (4 marks)

(d) Consider a hot water pipe inside a room at 25°C. The pipe is 5.0 m long and its outer radius is 12 cm; the surface temperature of the pipe is 120°C. Calculate the rate at which heat is radiated from the surface of the pipe if it (the pipe) has an emissivity of 0.90. (4 marks)

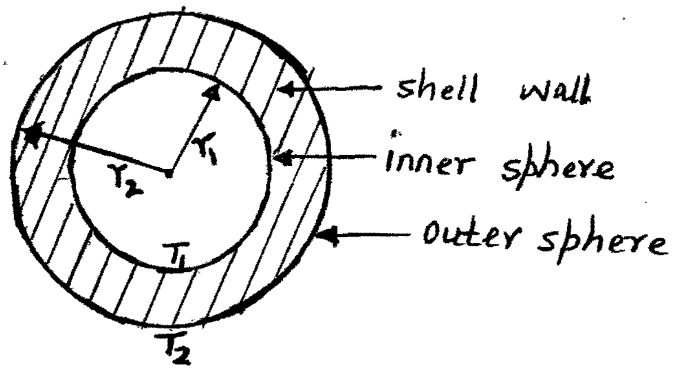

Fig. 2

QUESTION 3

(a) Derive the following expression which represents the work done by an ideal gas during a reversible adiabatic process:

$$W = \frac{1}{1-\gamma}\left[p_2V_2 - p_1V_1\right]$$

where 1 and 2 represent the initial and final states, respectively. (6 marks)

(b) Imagine air in a cylinder to behave like an ideal gas and that $\gamma=1.40$. The temperature of the air is 30°C. The air undergoes reversible adiabatic compression from an initial pressure of 1.0 atm and hence the volume decreases from 600 cm^3 to 100 cm^3.

Calculate the final temperature and pressure of the gas. (7 marks)

(c) One of the steps which represent the Otto cycle of a gasoline engine is based on reversible adiabatic compression. During this step, the pressure is found to increase from 1.0 atm to 20.0 atm.

By what factor does the temperature change during adiabatic compression if the gas is assumed to behave like an ideal diatomic gas? (6 marks)

(d) Consider an internal combustion engine with an efficiency of 45%. The engine operates between 500°C and 2000°C.

(i) Find the work done by the engine if the quantity of heat generated during combustion is 10^6 joules? (3 marks)

(ii) What is the maximum theoretical efficiency of the engine? (3 marks)

QUESTION 4

(a) In general, the average speed of a gas molecule is given by the following relationship:

$$\bar{v} = \frac{\int_0^\infty N(v) v dv}{N}$$

where

$$N(v) = 4\pi N \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

and the symbols have their usual meanings.

Show that the average speed of a gas molecule depends on the temperature of the gas and its mass as shown below.

$$\bar{v} = 1.59\sqrt{\frac{kT}{m}}$$

[**Hint:** $\int_0^\infty v^2 \exp(-\lambda v^2) dv = \frac{1}{2\lambda^2}$] (7 marks)

(b) Consider nitrogen molecules at a temperature of 25°C and a pressure of 1 atm. The diameter of a molecule of nitrogen is about 2 Å. Assume that its average speed is 600 ms^{-1} at this temperature. Consider the mean free path of the molecules to be

$$\lambda = \frac{1}{\pi d^2 n\sqrt{2}}$$, where the symbols have their usual meanings.

Calculate the mean-free-path and collision frequency of the molecules. (7 marks)

(c) The constant 'b' in the van der Waals equation of state, for carbon dioxide, is 4.3 x 10^{-5} m^3 mol^{-1}. Write down the equation. Assuming a carbon dioxide molecule is spherical, estimate its diameter. (4 marks)

(d) An ideal gas, initially at 500 K, absorbs 21.5 kJ of heat and expands isobarically at a pressure of 4 kPa. The volume of the gas increases from 2.0 m^3 to 4.0 m^3.

(i) Calculate the change in internal energy of the gas (4 marks)

(ii) Calculate the final temperature of the gas. (3 marks)

QUESTION 5

(a) Consider a p-V cycle PQRP. P to Q is an isovolumetric process, Q to R is an isothermal expansion, and R to P is an isobaric process.

(i) For each step of the cycle PQRP, determine whether the following quantities are positive, negative or zero: change in temperature, heat absorbed, work done by the gas and change in entropy of the gas. Take the gas to be the system. (12 marks)

(ii) Answer the same questions posed in (a)(i) above, for the whole cycle, i.e., P to Q to R to P. (3 marks)

(b) A heat reservoir at 150°C is used to evaporate 0.2 kg of water completely. The initial temperature of the water is 30°C.

(i) Calculate the minimum amount of energy required to evaporate the water completely. (6 marks)

(ii) What would be the change in entropy of the water? (4 marks)