

**UNIVERSITY OF SWAZILAND**

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**FACULTY OF SCIENCE**

**DEPARTMENT OF PHYSICS**

**SUPPLEMENTARY EXAMINATION 2012**

**TITLE OF PAPER : THERMODYNAMICS**

**COURSE NUMBER : P242**

**TIME ALLOWED : THREE HOURS**

**INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS**

**EACH QUESTION CARRIES 25 MARKS**

**MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.**

**THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.**

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### **INFORMATION**

For a monatomic gas:  $\gamma = \frac{5}{3}$  and  $C_v = \frac{3R}{2}$  ;  $C_p = \frac{5R}{2}$

For a diatomic gas:  $\gamma = \frac{7}{5}$

Universal gas constant,  $R = 8.31 \text{ Jmol}^{-1}\text{K}^{-1}$

Specific heat of water =  $4190 \text{ J kg}^{-1}\text{K}^{-1}$

Latent heat of vaporisation of water  $2.256 \times 10^6 \text{ J kg}^{-1}$

Stefan-Boltzmann constant =  $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$

### **QUESTION 1**

- (a) Consider an ideal gas which is caused to move through a p-V cycle ABCA. A to B is an isochoric process, B to C is an isothermal expansion, and C to A is an isobaric process.

Explain, for each leg or step of the cycle, whether each of the following variables is negative, zero or positive: *temperature change, heat, work done, internal energy change, entropy change*. Consider the gas to be the system. (15 marks)

- (b) A heat reservoir at 373 K is used to evaporate 0.2 kg of water originally at 313 K.
- (i) How much energy must flow from the reservoir to do this? (6 marks)
- (ii) By how much would the entropy of the water change? (4 marks)

## **QUESTION 2**

- (a) A rectangular container of dimensions 'a', 'b' and 'c' consists of N particles of an ideal gas. The mass of each particle is m and the density of the gas is  $\rho$ . The pressure exerted by the gas onto the area 'ab' is

$$p = \frac{m[(v_x^2)_1 + (v_x^2)_2 + \dots + (v_x^2)_N]}{abc}$$

Show that the root-mean-square speed of the particles is

$$v_{rms} = \sqrt{v^2} = \sqrt{\frac{3p}{\rho}} \quad (14 \text{ marks})$$

- (b) Measurements by spacecraft have shown that near the surface of the planet Venus the atmospheric pressure is 90 times that on Earth surface, and the temperature is 500°C, compared to typical temperatures on Earth of 10°C. What is the ratio of the speed of an average carbon dioxide molecule on Venus to that of an average carbon dioxide molecule on earth?

**Hint:**  $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$ , where the symbols have their usual meanings. (4 marks)

- (c) Consider air molecules at 0°C and 1 atmosphere pressure. The molecules collide with each other and the radius of an equivalent molecule is 2 Å. The mean free path of the molecules depends on their size as well as their concentration. For the conditions stated, the average speed of air molecules is about  $1 \times 10^7$  m/s and there are approximately  $3 \times 10^{25}$  molecules/m<sup>3</sup>.
- (i) Calculate the mean free path; (4 marks)
- (ii) Calculate the collision frequency. (3 marks)

### QUESTION 3

- (a) For Carnot cycle shown in Fig. 2.1, the ratio  $V_3/V_1$  is 15. Steps 1-2 and 3-4 represent isothermal processes whilst steps 2-3 and 4-1 stand for adiabatic processes. The temperature limits of the cycle are  $260^\circ\text{C}$  (step 1-2) and  $21^\circ\text{C}$  (step 3-4).

Determine the volume ratios of the isothermal and adiabatic processes, that is,  $V_4/V_1$  and  $V_3/V_4$ . Assume that the working medium is a diatomic gas. (10 marks)

- (b) Imagine a Carnot engine which takes 5000 J of heat during each cycle from the high-temperature reservoir at 300 K and gives out 3500 J to the low-temperature reservoir.
- (i) Calculate the temperature of the low-temperature reservoir. (5 marks)
- (ii) What is the thermal efficiency of the cycle? (3 marks)
- (c) Consider the temperatures of the hot and cold reservoirs of a real engine to be 600 K and 400 K, respectively.
- (i) If the real engine is replaced by a Carnot engine working between the same two temperatures, with an input  $Q_H$  of 20 kJ, how much heat would be rejected and how much work would the engine do? (5 marks)
- (ii) What would be the co-efficient of performance of the Carnot engine if it were used as a refrigerator between the same two temperatures? (2 marks)

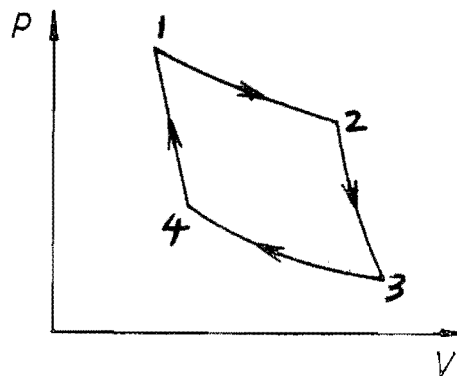


Fig. 2.1

#### **QUESTION 4**

- (a) Use the ideal gas law and the relationship  $pV^\gamma = \text{constant}$  to show that for an adiabatic process  $TV^{\gamma-1} = \text{constant}$  and that  $T^\gamma p^{1-\gamma} = \text{constant}$ , where the symbols have their usual meanings. (7 marks)
- (b) Two thousand moles of a monatomic ideal gas is taken through the following cyclic process (1) An isobaric expansion from  $2 \text{ m}^3$  to  $4.6 \text{ m}^3$  at a pressure of  $4 \times 10^6 \text{ Nm}^{-2}$ ; (2) An isochoric decrease in pressure from  $4 \times 10^6 \text{ Nm}^{-2}$  to  $1 \times 10^6 \text{ Nm}^{-2}$ ; (3) An adiabatic compression back to the initial state.
- (i) Sketch the p-V diagram for this cyclic process; (2 marks)
- (ii) Find the work done during each step of the cycle and the net work done for the cycle. (9 marks)
- (iii) Calculate the heat exchanged during each step of the cycle and the net heat for the cycle? (7 marks)

### **QUESTION 5**

- (a) Consider a water pipe of internal radius  $x$ , external radius  $y$  and length  $z$ . The inside temperature is  $T_1$  while the surroundings are at a temperature of  $T_2$  (where  $T_1 > T_2$ ). Show that heat is conducted through the walls of the pipe at the rate

$$\frac{dQ}{dt} = \frac{2\pi k(T_1 - T_2)z}{\ln\left(\frac{y}{x}\right)}$$

[ $k$  is the thermal conductivity of the pipe].

(12 marks)

- (b) The surface temperature of a spherical mass of molten metal (the source), 3.0 m in radius, is 1073 K. It is surrounded by a spherical shell with an inside radius of 3.0 m and an outside radius of 6.0 m. The thermal conductivity of the shell is  $42 \text{ Wm}^{-1}\text{K}^{-1}$ . If the outside of this spherical shell is exposed to room temperature (298K), what is the rate at which heat flows through the shell?

(7 marks)

- (c) Assume that the total surface area of the human body is  $2.0 \text{ m}^2$  and that the temperature at the surface of the body is  $40^\circ\text{C}$ . How much heat would be lost from the body in 10 min if it is exposed to an environment at  $15^\circ\text{C}$ ? Assume that the emissivity of the skin is 0.80?

(6 marks)