UNIVERSITY OF SWAZILAND

153

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2011/2012

TITLE OF PAPER: QUANTUM MECHANICS

COURSE NUMBER: P342

TIME ALLOWED : THREE HOURS

THERE ARE **FIVE** QUESTIONS IN THIS PAPER. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question one

(A) (i) State the principle of simultaneity in relativity. (2 marks)

(ii) Write down the Lorentz transformation equations relating the coordinates of an event in two different inertial frames of reference moving with relative velocity along the x-axis. (4 marks)

(iii) In a certain inertial frame of reference two light pulses are emitted, a distance 5 km apart and separated by 5 microseconds. An observer, who is traveling parallel to the line joining the points where the pulses are emitted, notes that the pulses are simultaneous. Find the velocity of the observer with respect to the inertial frame. (5 marks)

(B) (i) Given two observers O and O' with O' moving at uniform velocity 'v' in the positive x-direction relative to O, use the appropriate Lorentz transformation equations to show that

$$U_x' = \frac{U_x - v}{1 - vU_x / c^2}$$

where U_X' and U_X are the velocity components of a body along the x-direction in O' and O respectively. (5 marks)

(ii) A spaceship moving away from the Earth at a velocity 0.75 c with respect to the Earth, launches a rocket in the direction away from the Earth that attains a velocity 0.75 c with respect to the spaceship. What is the velocity of the rocket with respect to the Earth?
What would be your result if solved classically. Comment
(6 marks)

(C) A spacecraft moves at a speed of 0.9c If its length is Lo when measured from inside the spacecraft, what is its length measured by a ground observer?
(3 marks)

Question Two

(A) What was de Broglie hypothesis. (2 marks)

(B) $E = h\nu$ and $p = h / \lambda$ are the familiar expressions for energy and momentum of a photon What are the corresponding expressions for particle waves? (2 marks)

(C) Show that the energy of a particle can be expressed in terms of the wave number 'k' as $\hbar^2 k^2 / 2m$ (2marks)

(D) Distinguish between phase velocity and group velocity with their corresponding equations. (4 marks)

(E) Derive expressions for phase velocity and group velocity of matter waves. Which of these is more meaningful to represent the motion of a quantum particle? Why? (5 marks)

(F) (i) What is the significance of Einstein's experiment on the photoelectric effect. (1 mark)

(ii) Light of wavelength 5000 Å is incident on a metal surface having work function of 1.9 eV. The stopping potential for the emitted electron is 0.4 V.

Calculate:

(a) The energy of a photon in eV (2 marks)
(b) The maximum energy of the photoelectron (2 marks)
(c) The threshold frequency (2 marks)

(G) (i) State Heisenberg uncertainty principle. (2marks)

(ii) An electron absorbs a certain amount of energy and reemits it within 10^{-8}s. What is the uncertainty in its energy before reemitting? (1 mark)

Question Three

(A) The wave function representing a quantum particle is

$$\psi(x,t) = \int_{-\infty}^{\infty} Ce^{-a^2(k-k_0)^2} e^{i(kx-\omega t)} dk$$

Given that $\psi(x,0) = \frac{C\sqrt{\pi}}{a} e^{-x^2/4a^2} e^{k_0 x}$

(i) Find the half-width Δk and Δx (8 marks)
(ii) Find the product $\Delta k.\Delta x$ (2 marks)
(iii) With appropriate substitution, find the value $\Delta x\ .\Delta p$. (2 marks)
(iv) State the significance of the above expression in quantum mechanics. (2 marks)

[Note: take half width as $e^{-1/2}$ of the maximum value]

(B) (i) One of the conditions that a wave function is acceptable to represent quantum particle is that it should be normalisable. State this condition mathematically. (2 marks)

(ii) Normalize the wave function $\psi = A\sin\frac{n\pi}{a}x$ of a particle confined within length 'a'. (5 marks)

(iii) Write down the condition for two wave functions Ψ_m and Ψ_n to be orthogonal. (2 marks)
What is the physical significance of the above condition? (2 marks)

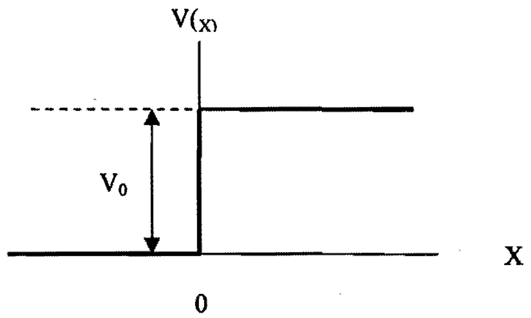
Question four

(A) Write down the time dependent Schrödinger wave equation in one dimension. (2 marks)

(B) Show how the time independent Schrödinger wave equation can be obtained from the time dependent Schrödinger wave equation. (5 marks)

(C) What is the physical meaning of $|\Psi(x,t)|^2$? (2 marks)

(D) A beam of particles travelling along the positive x- direction encounters a potential step $V(x) = V_0$ for $x < 0$ and $V(x) = 0$ for $x > 0$ as shown in figure below. (Assume total energy $E > V_0$).



(i) By solving Schrödinger wave equation for the two regions, find the amplitudes of the reflection and transmission of the particles across the step. (10 marks)

(ii) Find expressions for the reflection coefficient R and transmission coefficient T. (2 marks)

(iii) Show that $R + T = 1$. (2 marks)

(iv) Compare your results with the classical case. (2 marks)

Question Five

(A) Write down the expressions for the quantum mechanical operators corresponding to the following classical variables:

(i) Position 'x'
(ii) Linear momentum 'P_x'
(iii) Time't'
(iv) Total energy 'E' (4 marks)

(B) Obtain the total energy operator for a free particle moving along x- direction. (4 marks)

(C) (i) Write down the commutative law for two operators A and B. (2 marks)

(ii) What is the physical significance if two operators do not commute? (2 marks)

(iii) Verify whether or not operators 'x ' and 'd /dx' commute. (3 marks)

(D) (i) State what is meant by an eigenfunction. (2 marks)

(ii) Define a Hermitian operator. (2 marks)

(iii) Prove that eignen values of Hermitian operators are real. (4 marks)

(iv) Find the eigenvalue of the operator d /dx operating on function e^{ax}. (2 marks)