UNIVERSITY OF SWAZILAND 159

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2011/2012

TITLE OF PAPER: QUANTUM MECHANICS

COURSE NUMBER: P342

TIME ALLOWED : THREE HOURS

THERE ARE **FIVE** QUESTIONS IN THIS PAPER. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(A) (i) What is meant by an inertial frame of reference? (2 marks)

(ii) State the two postulates of the special theory of relativity. (4 marks)

(iii) State the principle of simultaneity in relativity (3 marks)

(iv) Write down the Lorentz transformation equations relating the co-ordinates of an event taking place in two different inertial frames moving along the x-axis with a relative velocity 'v' (4 marks)

(v) Newton's laws of motion are unaffected by Galilean transformation . Why then there is need for Lorentz transformation to treat relativity? (2 marks)

(B) (i) The life-time of a certain particle at rest in the earth's frame is 2.2×10^{-6} s. What will its life-time when it is travelling at a speed of 0.98 c relative to the earth? (3 marks)

(ii) Two space ships travel at 0.99c in opposite directions relative to an outside observer. Calculate their relative velocity observed in either space ship ,

1.relativistically

2. classically

Comment on the results. (5 marks)

(iii) A rod is at rest along the X- axis in a reference frame S. According to an observer in another frame S', moving at 0.5 c the length of the rod is 0.75 m. What is the length of the rod according to the observer in frame S? (2 marks)

Question two **3**

(A) (i) Calculate the wavelength of neutrons having 38.8 MeV energy. (4 marks)

(ii) State why neutrons can be diffracted by ca crystalline solids. (2 marks)

(B) (i) State and explain the important features of Einstein's experiment on photoelectric effect. (6marks)

(ii) In a photoelectric effect experiment, light of wavelength 5500 Å is incident on a metal surface. The stopping potential for the emitted electron is 0.42 V. Calculate:

1. The maximum energy of the photoelectron (2 marks)
2. The work function of the metal and (3 marks)
3. The threshold frequency (2 marks)

(C) (i) State Heisenberg uncertainty principle. (2 marks)

(ii) The speed of a body of mass 3000 kg can be measured with an accuracy of 10^{-3} ms^{-1} and its position with an accuracy of 16 m. Is Heisenberg's uncertainty principle applicable here? Why? (4 marks)

Question Three

(A) (i) State any two properties of an acceptable wave function in quantum mechanics. (4 marks)

(ii) Distinguish between phase velocity and group velocity of a wave packet. (2 marks)

(iii) Given that the momentum of a classical particle p = mv, show that the group velocity represents a wave packet (i.e. $v_g = v$). (4 marks)

(B) The wave function of a particle is $\psi(x) = A\, e^{-ax}$ where a >0.

(i) Normalise the above wave function (9 marks)

(ii) Find the interval from the origin such that the probability of finding the particle in this interval is 50%

$$\int_0^{\infty} e^{-ax} dx = \frac{1}{a}$$ (6 marks)

Question Four

Consider a particle confined in a box having potential of the form

$$V(x) = 0 \, for 0 \leq x \leq L$$
$$V(x) = \infty \, elsewhere$$

(A) Set up the time-independent Schrodinger wave equation for the particle. (2 marks)

(B) Solve the above equation for all values of x and show that the particle has discrete energy

$$E_n = \frac{n^2 h^2}{8mL^2}, \quad \text{where n = 1,2,3 ...}$$ (8 marks)

(C) Normalise the wave function obtained in (B) above. (6 marks)

(D) Sketch the wave function for n = 2 and n = 3. (3 marks)

(E) Show that the expectation value of the position of the particle is $< x > = L / 2$. Comment on this result. (6 marks)

Question Five

(A) (i) State what is meant by a Hermitian operator in quantum mechanics. (3 marks)

(ii) Show that the operator $ai\frac{d}{dx}$ is Hermitian operator, where 'a' is a constant. (6 marks)

(iii) State the commutation rule for two operators A and B (2 marks)

(iv) Do the momentum and position operators $P_x = -i\hbar\, d/dx$ and $x = x$ commute? Verify this. Comment on your result. (4 marks)

(B) (i) The classical expression for angular momentum is **L = r X P.** Obtain the corresponding quantum mechanical expression for the angular momentum operator. (4 marks)

(ii) Show that any two components of the angular momentum are not compatible observables. (6 marks)