UNIVERSITY OF SWAZILAND.

FACULTY OF SCIENCE.

DEPARTMENT OF PHYSICS.

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MAIN EXAMINATION 2011 / 2012

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412.

TIME ALLOWED : THREE HOURS.

ANSWER ANY FOUR QUESTIONS . ALL CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One.

(a) (i) Draw conventional unit cells of face - centred and body - centred cubic lattices of lattice constant 'a'. For each lattice, write down the number of lattice points per cell and the volume of the primitive cell. (2 + 4 marks)

(ii) What is meant by "packing fraction" of a crystal?
Determine the packing fraction of the diamond structure.
(The body diagonal of its unit cell is 8R, where R is the atomic radius).
(2 + 4 marks)

(b) (i) In the diagram of a cubic unit cell, show a (121) and a (212) plane. (4 marks)

(ii) Calculate the separation between two (123) planes of an orthorhombic cell with unit cell dimensions a = 0.82 nm, b = 0.94 nm and c = 0.75 nm (4 marks)

(c) A first order reflection from the (111) planes of a cubic crystal was observed at a glancing angle of 11.2^0 when x-rays of wavelength 154 pm were used. Calculate the length of the side of each cell. (5 marks)

Question Two.

(a) (i) What is van der Waals -London attractive interaction in inert gas crystals? (5 marks)

(ii) Explain how Pauli's exclusion principle is responsible for the repulsive interaction in inert gas crystals. (5 marks)

(b) (i) Derive the Bragg law $2d\sin\theta = n\lambda$ for diffraction of waves by a crystal lattice. (5 marks)

(ii) Explain why visible light cannot be used for Bragg reflection experiments. (2 marks)

(iii) In the X-ray photograph of a cubic lattice, lines are observed at the following Bragg angles in degrees: 12.3, 14.1, 20.2, 24.0, 25.1, 29.3, 32.2 and 33.1. Assign Miller indices to these lines and identify the lattice type. (8 marks)

Question Three

(a) Derive the phonon dispersion relation $\omega = \left(\frac{4C}{M}\right)^{1/2} \sin\frac{1}{2}ka$ for a one-dimensional monatomic linear lattice of lattice constant 'a', atomic mass 'M' and force constant 'C'.

(13 marks)

(b) (i) Draw a sketch showing how the phonon frequency varies with wave vector in the first Brillouin zone. (3 marks)

(ii) What are the values of the frequency for k =0 and k= π / a ?

(2 marks)

(i) Show that when the phonon wavelength is large compared to the interatomic spacing, the phase velocity, $\frac{\omega}{k} = a\sqrt{\frac{C}{M}}$, where the symbols have the usual meanings. (4 marks)

(c) Calculate the velocity of the elastic waves in a linear lattice of lattice constant 1Å, force constant 30 Nm^{-1}, and atomic mass 10^{-27} kg.

(3 marks)

Question Four

(a) (i) Explain how an intrinsic sample of silicon can be made n -type or p - type by appropriate doping. (4 marks)

(ii) State what is meant by **effective density of states** of a semiconductor. (2 marks)

(iii) The effective density of states in the conduction and valence bands of a semiconductor is given as:

$$N_{c,v} = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2}$$

where the symbols have the usual meanings. Using the above expression write down the electron and hole concentrations in the conduction and valence bands. (2 marks)

(iv) Show that the electrical conductivity of an intrinsic semiconductor can be expressed as

$$\sigma_i = A\exp\left(\frac{-E_g}{2kT}\right),$$

where E_g is its band gap and other symbols have the usual meanings. (4 marks)

(v) Explain how would you use the above expression to find the band gap of a material experimentally. (4 marks)

(b) Calculate: (i) the effective density of states and (6 marks)
(ii) the intrinsic carrier concentration of silicon. (3 marks)

[Effective masses of electrons and holes are 1.1 m_0 and 0.56 m_0, respectively. Band gap of silicon = 1.1 eV]

Question Five

(a) Discuss briefly the free electron approximation in metals. (4 marks)

(b) Assume a plane wave $\psi_k(r) = \exp i(k.r)$, where symbols have the usual meanings, representing a free electron. Use the Schrodinger wave equation to obtain its energy eigenvalues, ϵ_k. (4 marks)

(c) (i) What is meant by Fermi energy? (2 marks)

(ii) Use the results in (b) above to show how the Fermi energy is related to the electron concentration, and hence derive an expression for the density of states of the electrons in a metal.

(10 marks)

(d) Calculate the Fermi energy of potassium, given that it has a density of $8.6\text{x}10^{2}$ kg m^{-3} and an atomic weight of 39. (5 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27\ kg}$
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ l\ mol^{-1}$