UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2011/2012

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

THIS PAPER CONTAINS **FIVE** QUESTIONS. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER CONTAINS **EIGHT** PAGES INCLUDING THE COVER PAGE.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) (i) Explain the difference between a macrostate and a microstate of a system of particles. (3marks)

(ii) Define the term "thermodynamic probability" in statistical physics. (2 marks)

(iii) What is the significance of the term "thermodynamic probability" as regards the properties of the system? (2 marks)

(iv) State the relationship between thermodynamic probability 'W' and entropy 'S' of a system. (2 marks)

(v) What is meant by degeneracy of an energy level? (2 marks)

(vi) Two distinguishable particles are distributed among three non-degenerate energy levels having energy 0, ϵ and 2ϵ, such that the total energy is always 2ϵ. What is the entropy of the system? (4 marks)

(b) (i) Define 'density of states' of a system. (3 marks)

(ii) Write down the expression for the density of states for a system of fermions. (2 marks)

(iii) Calculate the density of states of potassium ($[Ar]4s^1$) of volume 10^{-4} m^3 at the Fermi level of 3.0×10^{-19} J. (5 marks)

Question Two

(a) The distribution function for a system of classical particles in equilibrium at temperature T is given as: $n_s = g_s e^{\alpha + \beta \varepsilon_s}$, where the symbols have their usual meanings.
Show that the numerical value of the constant β in this equation is equal to - 1/(kT), where k is the Boltzmann constant.

(10 marks)

(b) Derive the following expressions for an ideal gas in terms of its partition function 'Z'.

(i) The total energy E = $NkT^2\left(\frac{\partial \ln Z}{\partial T}\right)_V$ (6 marks)

(ii) Pressure P = $NkT\left(\frac{\partial \ln Z}{\partial V}\right)_T$ (3 marks)

(c) Given that the partition function of a classical system $Z = aVT^4$, where 'a' is a constant, find the values of

(i) total energy

(ii) pressure (6 marks)

Question Three

(a) (i) Distinguish between an extensive and an intensive variable used in thermodynamics. Give at least one example of each. (4 marks)

(ii) Two systems 1 and 2, having identical particles, have entropies S_1 and S_2 and statistical weights W_1 and W_2. When the two systems are mixed together, what is:

1. The total entropy S_T (2 marks)

2. The total statistical weight W_T of the mixture? (2 marks)

(iii) Do the above results for total entropy and total weight agree with the equation that links entropy and thermodynamic probability? Explain. (3 marks)

(b) Derive an expression for the entropy 'S' of a system in terms of its partition function 'Z'. (10 marks)

$$Given: W = N!\prod_{S}\left\{\frac{g_s^{\,n_s}}{n_S!}\right\}$$

(c) Show that the partition function of a perfect classical gas can be expressed as:

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$ (4 marks)

[See appendix 1 for definite integrals]

Question Four

(a) Use the Bose-Einstein distribution function of an assembly of identical non-interacting particles in thermal equilibrium to derive the Planck's radiation law for spectral distribution of energy radiated from a constant temperature enclosure.
(9 marks)

(b) Obtain an expression for the total energy per unit volume emitted from the enclosure at temperature T. (5 marks)

(c) (i) State briefly, the meaning of ***Bose - Einstein condensation.*** (3 marks)

(ii) The density of an ideal gas consisting of particles having a mass of 6.65×10^{-27} kg is 1.17×10^{26} m^{-3}.

1. Calculate the Bose temperature T_B of the gas. (5 marks)

2. What fraction of the particles will be in the ground state at a temperature of $0.1T_B$. (3 marks)

Given:

$$N = 2.612V\left(\frac{2\pi mkT_B}{h^2}\right)^{3/2}$$

Question Five

(a) The Fermi function of a system is given as: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/kT}+1}$ where the symbols have their usual meanings. Obtain its values at absolute zero, for the cases $\epsilon > \epsilon_F$ and $\epsilon < \epsilon_F$. What is the physical meaning of these results? (6 marks)

(b) The Fermi level of a solid is 8.6 eV. Find the probability of occupation of an electron:

(i) in an energy level 0.1 eV above the Fermi level at 300K and at 400K.

(ii) in an energy level 1.0 eV above the Fermi level at 300K and at 400K.

Comment on the results. (6 marks)

(c) Derive an expression for the paramagnetic susceptibility of a metal to show that it is independent of temperature. Draw diagrams where necessary.

[Neglect the response to the applied field due to the orbital motion of the electrons. Assume energy $\mu_B B \ll \epsilon_F$ and that ϵ_F is a constant for the material]

(13 marks)

Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty e^{-ax}dx = \frac{1}{a},(a > 0)$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Planck's constant	h	6.63×10^{-34} J.s
Stefan - Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of ${}_2^4$He atom		6.65×10^{-27} kg
Mass of ${}_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ L\ mol^{-1}$