204

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2011/2012

TITLE OF THE PAPER: COMPUTATIONAL METHODS-II

COURSE NUMBER: P482

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **30** MARKS.
- **SECTION B** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **70** MARKS.

Answer **any** two questions from **section A** and
all the questions from **section B**.
Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

Section A

Question 1

(a) Consider the statistical experiment of flipping an unbiased coin. Write down a maple code that generates a list that contains the outcome of N flips of the unbiased coin using the Maple random number generator. Define your symbols appropriately.

[4 mark]

(b) Consider a random walker in two dimensions on a square lattice. The walker starts at the origin (0,0) and moves non-stop for M steps.

(i) Write an algorithm to simulate the movement of the random walker. Assume the random walker takes a unit step at each instant and that he/she moves up or down, left or right with equal probabilities.

[6 marks]

(ii) Explain how would you utilize this program to show that the mean square displacement of a random walker is proportional to the duration of the walk.

[3 marks]

(iii) How does the process of random walker relates to the theory of diffusion?

[2 marks]

Question 2

(a) Use the Euler method to solve

$$\frac{dN(t)}{dt} = -N(t)$$

with $N(0) = 1$.

(i) Determine N(t_i) after i steps of size Δt. What is the exact solution?

[4 marks]

(ii) For $\Delta t = 0.5, 1.5, 3$ calculate $N(t_i)$ for $i = 0...4$. Sketch $N(t_i)$ vs t_i. Which values of Δt is the method stable.

[5 marks]

(b) Write a difference formula corresponding to the following equations

(i) $\frac{\partial}{\partial t}\rho(x,t) = \frac{\partial^2}{\partial x^2}\rho(x,t) + h\rho(x,t)$

[3 marks]

(ii) $\frac{d}{dt}v(t) = gy(t) + A\sin(\omega t)$

[3 marks]

Question 3

(a) A continuous voltage signal $V(t)$ is measured at a time step of Δt for N times.

(i) What is the period T of the measurement.

[1 mark]

(ii) Given that the signal is periodic, what is the highest frequency of the signal can be resolved from the recorded data. Support your answer.

[3 mark]

(b) Sketch the power spectrum of the following signals

(i) $D_1(t) = 1.0 + \sin(t)$

(ii) $D_2(t) = 3\sin(t) + 2\sin(3t) + \sin(5t)$

(iii) $D_3(t) = \sin^2(t)$

[6 marks]

(c) Write a Maple script that constructs the signal

$$g(t_i) = 1.0 + \exp(2\pi t_i) + r_i$$

where $t_i = i \cdot \Delta t$ for $i = 0..L$ and r_i is a noise term that can be generate using uniformly distributed random numbers with an amplitude $|r_i| = 0.1$.

[5 marks]

Section B

Question 4

A radioactive sample contains N_0 radioactive nuclei at $t = 0$. The decay rate of the radioactive nuclei is $\lambda = 0.27 \times 10^6 s^{-1}$. Imagine that the activity of the sample was measure from $t = 0$ upto $20\mu s$ at time intervals of $1\mu s$. Utilize the pseudo-code below to simulate the radioactive decay experiment and to answer the questions below.

(a) Using your simulation show that for a large number N_0 the number of radioactive nuclei $N(t)$ decays exponential with time.

[15 marks]

(b) Compare plots of $\ln N(t)$ versus t with different values of N_0. Explain your observations.

[10 marks]

(c) Explain in your own words how a process that is spontaneous and random at its very heart can lead to exponential decay.

[5 marks]

A pseudocode for simulating the decay process is simple:

Initialization of parameters:

N[0] ! the initial number of radioactive nuclei.

λ *! decay rate*

dt ! discrete time step, for each interval dt we count the number of nuclei that have decay.

$p := \lambda * dt$ *!probability that a radioactive nucleus after a timestep dt.*

$T = Period/dt$ *! number of measurement conducted (number of iterations)*

Implementation stage:

```
> for j from 0 to T − 1 do
> if (N[j] > 0) then # the simulation stops when all have decayed.
> dN[j]:=0;
> for i from 1 to N[j] do
> r[i]:=stats[random,uniform]();
> if (r[i] < p) then
> dN[j] :=dN[j] +1;
```

```
# number of decays at given timestep. > end if:
> end do;
> N[j+1]:=N[j] - dN[j];
> end if;
> end do;
```

Outputs:

$[t, N[t]]$, $[t, ln(N[t])]$, *etc...!*

Question 5

The dynamics of a charged particle in a magnetic field is described by Newton's second law:

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{v} \times \mathbf{B} - \frac{\gamma}{m}\mathbf{v}$$

where **B** is the magnetic field, γ represent the coefficient of a damping force, m and q corresponds to the mass and the charge of the particle, respectively.

(a) **Uniform field.** When **B** is in the z-direction, then the dynamics are given by four equation:

$$\begin{aligned} \frac{dx(t)}{dt} &= v_x(t) \\ \frac{dy(t)}{dt} &= v_y(t) \\ \frac{dv_x(t)}{dt} &= \frac{qB}{m}v_y(t) - \frac{\gamma}{m}v_x(t) \\ \frac{dv_y(t)}{dt} &= -\frac{qB}{m}v_x(t) - \frac{\gamma}{m}v_y(t) \end{aligned}$$

(i) Write a program to simulate the dynamics of the charged particle. Assume that the initial velocity of the particle is $\mathbf{v}(t = 0) = (1, 0, 0)$ and the initial position $\mathbf{r}(t = 0) = (0, 0)$. Let $dt = 0.001$, $qB/m = 2.0$, and $\gamma/m = 0$. Let the period of simulation T =10 (i.e. T/dt=10000). All this quantities are given in dimensionless units. Plot the trajectory of the particle [x(t),y(t)]. What is the shape of the trajectory?

[15 marks]

(ii) Choose an appropriate value(s) of γ/m to investigate the effects of the damping force. Plot the trajectory of the particle under the influence of a damping force with the same initial conditions as (i). Discuss your observations.

[10 marks]

(b) **Crossed Electric-Magnetic fields**. Now suppose an electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$ is introduced into the system . In this case we need to add an extra term in the equation describing the change in x-component of the velocity :

$$\frac{dv_x(t)}{dt} = \frac{qB}{m}v_y(t) - \frac{\gamma}{m}v_x(t) + \frac{qE}{m},$$

whilst the other equations remain the same. Assume that $qE/m = 2.0$, $qB/m = 2.0$, and $\gamma/m = 0$, in dimensionless units. Show that the particle released with an initial velocity $\mathbf{v}(t) = 0$ from the origin will move like a leaping frog along the y-axis. Explain why this is so.

[15 marks]