UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2012/2013

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR

QUESTION 1

(a) A stone is thrown downward from a height h with initial velocity v_s while at the same time a ball is projected vertically upward with an initial velocity $v_b = 20\text{m/s}$ from directly beneath such that the two collide at some height h'. At what time t later do the stone and the ball collide in terms of h, v_b, and v_s? **(7 marks)**

(b) Use a clear diagram as an aid to illustrate the two unit vector $\hat{r}$ and $\hat{\theta}$ in plane polar coordinates, and write down the equations of these unit vectors in terms of their Cartesian unit vector counterparts. **(6 marks)**

(c) Derive an expression for the time derivative of the unit vector $\hat{r}$ in plane polar coordinate, i.e. find $\frac{d\hat{r}}{dt}$. **(6 marks)**

(d) Use the volume element in spherical coordinates to find the volume of quarter hollow sphere with inner radius R_1 and outer radius R_2. **(6 marks)**

QUESTION 2

(a) What horizontal force F is to be applied to the cart shown in Figure 1 so that the blocks of masses m_1 and m_2 remain stationary with respect to the cart of mass m_3. Assume all surfaces are frictionless. The answer must be in terms of m_1, m_2, m_3 and g. **(6 marks)**

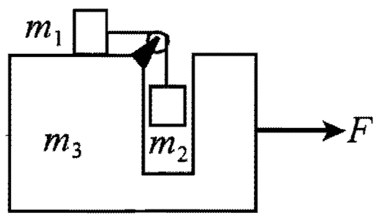

Figure 1.

(b) A particle of mass m is attached to a fixed point A on a vertical shaft by means of a light inextensible string of length $5l$. The particle is also connected to one end of a light rod of length $5l$. The other end of the rod is smoothly attached to point B which is $6l$ vertically below A. See Figure 2. The particle rotates in a horizontal circle with an angular velocity $\omega = \left(\frac{g}{10l}\right)^{1/2}$.

(i) Make a resolved force diagram for the mass m from which useful equations can be obtained. **(4 marks)**

(ii) Write down the equations of motion of the mass m based on the diagram made. **(2 marks)**

(iii) Find the tension in the string. **(7 marks)**

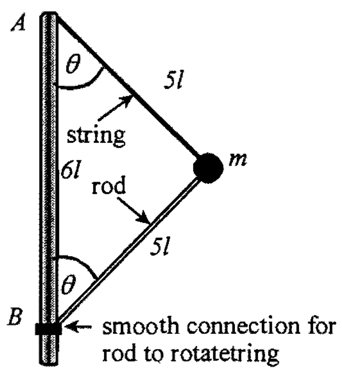

Figure 2.

(c) The International space station travels at speeds of around 27,685.7 km/h. The mass of the earth $M_E = 5.98 \times 10^{24}$ kg and the radius of the Earth is $R_E = 6.37 \times 10^6$m.

(i) Determine the radius of orbit of the space station from the centre of the earth. **(3 marks)**

(ii) How much time in minutes does it take to make one revolution around the earth? **(3 marks**

QUESTION 3

(a) Find the centre of mass of a thin triangle illustrated in Figure 3. **(9 marks)**

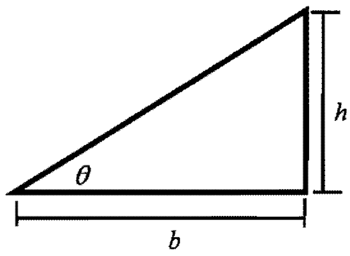

Figure 3.

(b) A rocket ascends from rest in a uniform gravitational field g by ejecting exhaust gases with constant speed u. The rocket is also retarded by air resistance $f = -bmv$, where b is a constant, and M and v are the instantaneous mass and velocity of the rocket, respectively. The rate at which the mass is expelled is $\frac{dm}{dt} = \gamma M, \gamma$ is a constant with appropriate unit.

(i) Develop the equation in the form of an integral that can enable you to determine the velocity of the rocket as a function of time. **(8 marks)**

(ii) Find the expression for the velocity. **(6 marks)**

(iii) Show that eventually the velocity becomes effectively constant, and find the terminal velocity. **(2 marks)**

QUESTION 4

(a) A particle of mass m moves from the lowest point inside a smooth spherical shell of inner radius R with an initial velocity v_0. as shown in Figure 4.

(i) Use the work-energy theorem to determine the normal force on the particle while it is still in contact with the inside surface of the sphere in terms of m, R, g, v_0 and θ. **(8 marks)**

(ii) Find an expression for the angle at which the particle leaves the circular path on the sphere and evaluate the angle for $v_0 = \sqrt{3RG}$. **(5 marks)**

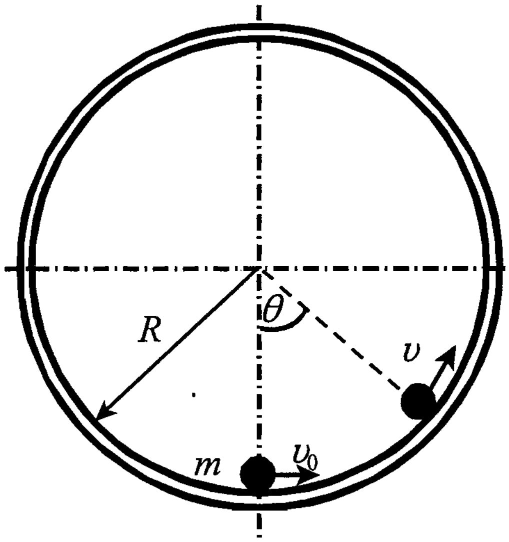

Figure 4.

(b) A particle is under the potential energy function:

$U(r) = \frac{r^3}{3} - \frac{r^2}{2} - 2r + 5.$

(i) Find the force acting on the particle. **(3 marks)**

(ii) Determine the equilibrium points. **(3 marks)**

(iii) Determine the stability of the equilibrium points. **(3 marks)**

(iv) Find the angular velocity of small oscillations about the stable equilibrium point. **(3 marks)**

QUESTION 5

(a) Show that the moment of inertia of a cylinder of radius R and mass m about is axis of symmetry is given by $I_c = \frac{1}{2}mR^2$. **(5 marks)**

(b) An unpowered spacecraft of mass m is launched towards a far off planet of mass M, at a distance b perpendicular to the line to the centre of the planet as shown in Figure 5. Find the distance b to enable the spacecraft to just graze the planet in terms of G, M, R and v_0. **(10 marks)**

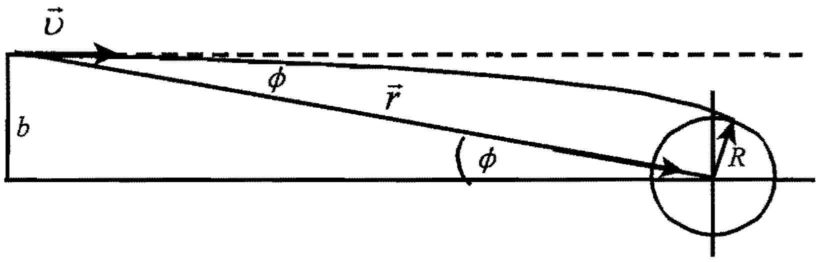

Figure 5

(c) A drum of radius R and mass M rolls down a plane inclined at an angle θ, as illustrated in Figure 6. Determine the acceleration of the drum down the inclined plane. **(10 marks)**

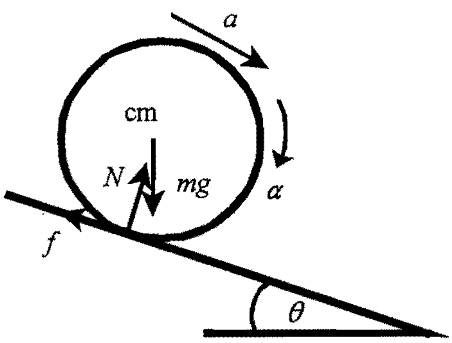

Figure 6.