UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION 2012/2013

TITLE OF PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

(a) Derive the basic kinematic equation, $v^2 = {v_0}^2 + 2a(x - x_0)$. **(4 marks)**

(b) A body is projected upward from the ground at an angle θ with the horizontal in a level field. It is discovered that the maximum height h reached by the body is equal to the range R. Find the angle θ. **(8 marks)**

(c) Use a clear diagram as an aid to illustrate how the position vector $\vec{r}$ is obtained in spherical coordinates, and write down the equation for this vector. **(7 marks)**

(d) Start with the infinitesimal volume element in spherical coordinates and find the volume of an eighth of a sphere of radius R. **(6 marks)**

QUESTION 2

(a) A 5 kg mass moves under the influence of a force $\vec{F} = (4t^2\hat{\imath} - 3\hat{\jmath})$ N. It starts from the origin at $t = 0$. Find

(i) its velocity $\vec{v}$, **(3 marks)**

(ii) its position $\vec{r}$ and, **(3 marks)**

(iii) $\vec{C} = \vec{r} \times \vec{v}$, for any later time. **(2 marks)**

(b) A particle of mass m slides without friction on the inside of a cone. The axis of the cone is vertical, and gravity is directed downward. The apex half angle of the cone is θ as shown in the Figure 1.

The path of the particle happens to be a circle in a horizontal plane. The tangential speed of the particle is v_0.

(i) Make a resolved force diagram for the mass m from which useful equations can be obtained. **(4 marks)**

(ii) Write down the equations of motion of the mass m based on the diagram made. **(2 marks)**

(iii) Find the radius of the circular path in terms of v_0, g, and θ. **(7 marks)**

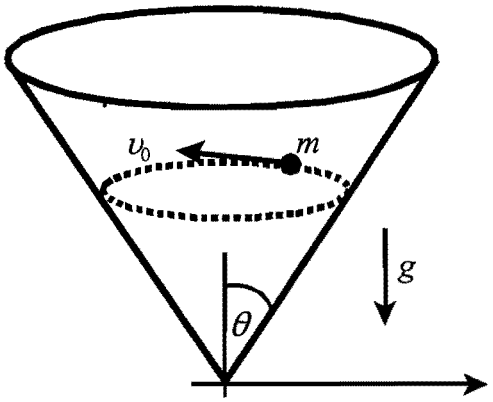

Figure 1.

(c) The International space station travels at speeds of around 27,685.7 km/h. The mass of the earth $M_E = 5.98 \times 10^{24}$ kg and the radius of the Earth is $R_E = 6.37 \times 10^6$m. Find the radius of orbit of the space station from the centre of the earth. **(4 marks)**

QUESTION 3

(a) An instrument carrying projectile accidentally explodes at the top of its trajectory. The horizontal distance between the launch point and the point of explosion is L. The projectile breaks into two pieces which fly apart horizontally. The larger piece has three times the mass of the smaller piece. The smaller piece returns to the launching point. How far away does the larger piece land? **(9 marks)**

(b) An empty freight car of mass M_0 starts from rest under an applied external force F. At the same time, sand starts to run into the car at a steady rate from a stationary hopper above. The sand has zero initial horizontal velocity when it strikes the freight car.

(i) Develop the equation in the form of an integral that can enable you to determine the velocity of the freight car as a function of time. **(8 marks)**

(ii) Find the expression for the velocity at alter times. **(6 marks)**

(iii) Determine whether the freight car reaches terminal velocity. **(2 marks)**

QUESTION 4

(a) A particle of mass *m* is attached to point *O* by an inextensible string of length l and negligible mass. The mass hangs below point *O*, when it is given an instantaneous horizontal velocity $u = 2\sqrt{gl}$,
where *g* is the gravitational acceleration. See Figure 2.

(i) At what angle θ does the mass deviate from a circular path? **(12 marks)**
(ii) What is the minimum initial velocity u to enable the mass to complete full circles in its motion? **(5 marks)**

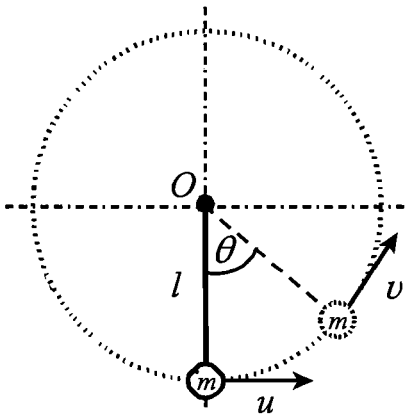

Figure 2.

(b) Consider a mass attached to a spring on a frictionless surface. Show that such a mass undergoes simple harmonic motion when disturbed by a distance x_0 from the equilibrium point. The force on the mass if $F = -kx$ and the velocity at x_0 is $v_0 = 0$. **(8 marks)**

Note: $\int \frac{dx}{\sqrt{a^2-x^2}} = arcsin\left(\frac{x}{a}\right) = \sin^{-1}\left(\frac{x}{a}\right)$.

QUESTION 5

(a) Find the moment of inertia of a uniform stick of mass *M* and length *L* about an axis through one of its ends. Use a diagram to illustrate the development of your solution. **(8 marks)**

(b) A particle moves along the x-axis with a velocity $\vec{v} = v\hat{\imath}$, and is acted upon by a frictional force $\vec{f} = -bv\hat{\imath}$, opposing the direction of motion. Using Figure 3, find

(i) the angular momentum at point *A*, **(2 marks)**

(ii) the torque at point *A*, **(2 marks)**

(iii) the angular momentum at point *B*, and **(2 marks)**

(iv) the torque at point *B*. **(2 marks)**

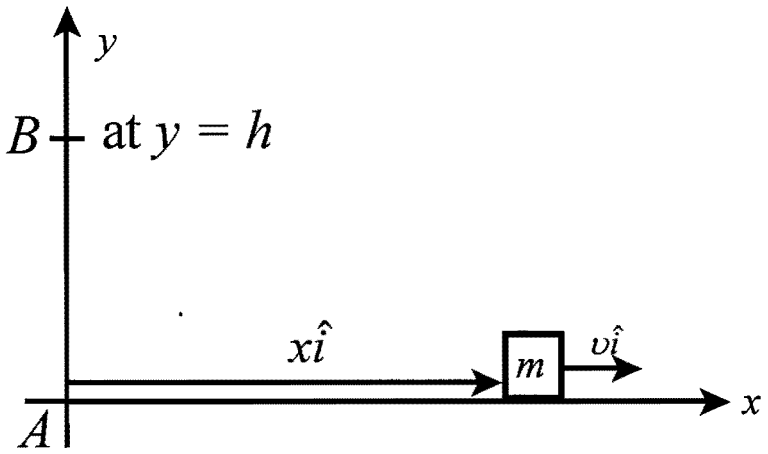

Figure 3.

(c) A body whose position vector is given by $\vec{r} = \left(at^3\hat{\imath} - bt^2\hat{\jmath} - ct\hat{k}\right)$ m is acted upon by a force $\vec{F} = \left(d\hat{\imath} + e\hat{\jmath} - f\hat{k}\right)$ N, where *a*, *b*, *c*, *d*, *e* and *f* are constants with appropriate units, and *t* is the time in seconds. Find the torque on and the angular momentum of the body, as functions of time. **(9 marks)**