UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS AND ENGINEERING

MAIN EXAMINATION 2012/203

TITLE OF PAPER: QUANTUM MECHANICS

COURSE NUMBER: P342

TIME ALLOWED : THREE HOURS

THERE ARE **FIVE** QUESTIONS IN THIS PAPER. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question one

(A) (i) What is meant by an inertial frame of reference? (2 marks)

(ii) State the two postulates of the special theory of relativity. (4 marks)

(iii) Michaelson - Morely experiment was point of departure from classical to relativistic physics.
What was the aim of this experiment?
What was the result?
What was the correct interpretation of the result given by Einstein?
(5 marks)

(iv) Newton's laws of motion are unaffected by Galilean transformation. Why then there is need for Lorentz transformation to treat relativity?
(2 marks)

(B) (i) Write down the Lorentz transformation equations relating the coordinates of an event in two different inertial frames of reference moving with relative velocity along the x-axis. (4 marks)

(ii) Two space ships travel at 0.99c in opposite directions relative to an outside observer. Calculate their relative velocity observed in either space ship,

1. relativistically and
2. classically (3+1 marks)

Comment on the results. (1 mark)

(iii) A rod is at rest along the X- axis in a reference frame S. According to an observer in another frame S', moving with velocity 0.5c along XX' axes, the length of the rod is 0.75 m. What is the length of the rod according to an observer at rest in frame S? (2 marks)
What would be the length of the same rod according to observer in S, if the rod were at rest in the moving frame S'? (1 mark)

Question two

(A) (i) Einstein's explanation of the photoelectric experiment proved the concept of quantum nature of electromagnetic radiation.
Describe briefly the experiment with the help of a sketch of the set up.
(4 marks)
State and explain the main features of the experiment. (8 marks)

(ii) In a photoelectric effect experiment, light of wavelength 5500 Å is incident on a metal surface. The stopping potential for the emitted electron is 0.42 V. Calculate:

1. The maximum energy of the photoelectron (1 mark)
2. The work function of the metal and (2 marks)
3. The threshold frequency (2 marks)

(B) (i) State the de Broglie hypothesis regarding the wave nature of particles.
(2 marks)

(ii) The Davisson -Germer experiment on diffraction of electrons by a crystal was a proof of de Broglie hypothesis. In one experiment, they used electrons having energy 54 eV and obtained the wavelength of electron as 1.65×10^{-10} m. Show by calculation how far this value agrees with the de Broglie hypothesis.
(3 marks)

(iii) Calculate the de Broglie wavelength associated with a 100 g bullet moving at 900 ms^{-1}. Compare it with your result in (ii) above and comment.
(3 marks)

Question three

(A) (i) $\psi(x,t)$ is the wave function of a quantum particle. State what each of the following represent.

$|\psi(x,t)|^2$

$|\psi(x,t)|^2 dx$ (4 marks)

(ii) State the properties of the wave function so that the above representations are meaningful.

(3 marks)

(B) Wave function in position of a quantum particle is gives as

$$\psi(x) = \left(\frac{1}{2\pi}\right)^{1/4} \frac{1}{\sigma} \exp\left(\frac{-x^2}{4\sigma^2}\right).$$

(i) Find the expectation value <x>. (3 marks)

(ii) Find the uncertainty Δx in the particle's position. (3 marks)

(iii) Obtain the momentum wave function of $\psi(x)$. (3 marks)

(iv) Find the expectation value <p>. (3 marks)

(v) Find the uncertainty in the particle's momentum Δp. (3 marks)

(vi) Find the product Δx.Δp. (1 marks)

(vii) Comment on the above result. (2 marks)

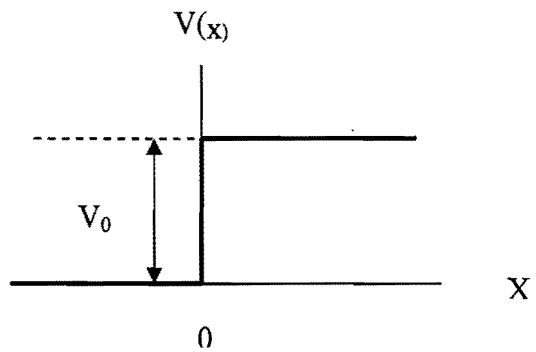
(Hint: get the Fourier transform using the definite integrals given in appendix)

Question four

(A) Write down the time dependent Schrödinger wave equation in one dimension. (2 marks)

(B) Show how the time independent Schrödinger wave equation can be obtained from the time dependent Schrödinger wave equation. (5 marks)

(C) A beam of particles travelling along the positive x- direction encounters a potential step $V(x) = V_0$ for $x > 0$ and $V(x) = 0$ for $x < 0$ as shown in figure below. (Assume total energy $E > V_0$).



(i) By solving Schrödinger wave equation for the two regions, find the amplitudes of the reflection and transmission of the particles across the step. (10 marks)

(ii) Find expressions for the reflection coefficient R and transmission coefficient T. (4 marks)

Show that $R + T = 1$. (2 marks)

Compare your results with the classical case. (2 marks)

Question five

(A) (i) State what is meant by a Hermitian operator in quantum mechanics. (2 marks)

(ii) Show that the operator $ai\frac{d}{dx}$ is Hermitian operator, where 'a' is a constant.

(4 marks)

(iii) Prove that eignenvalues of Hermitian operators are real. (4 marks)

(iv) State the commutation rule for two operators $\widehat{A}$ and $\widehat{B}$. (2 marks)

(v) Verify whether or not the momentum and position operators $\widehat{P} = -i\hbar\frac{d}{dx}$ and $\widehat{x} = x$ commute.
Comment on your result. (3 +1 marks)

(B) (i) The classical expression for angular momentum is **L = r X p.** Obtain the corresponding quantum mechanical expression for the angular momentum operator.

(4 marks)

(ii) Show that any two components of the angular momentum are not compatible observables. (5 marks)

Appendix 1

Definite integrals

$$\int_0^\infty e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}x^3dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}x^5dx = \frac{1}{a^3}$$

$$\int_0^\infty e^{-ax^2}xdx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^2dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty x^{1/2}e^{-\lambda x}dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty x^4e^{ax^2}dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty \frac{x^4e^x}{(e^x-1)^2}dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^3}{(e^x-1)}dx = \frac{\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{(e^x-1)}dx = \frac{2.61\pi^{1/2}}{2}$$

$$\int_0^\infty e^{-ax}dx = \frac{1}{a},(a > 0)$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{2a^2}\right)}dx = a\sqrt{2\pi}$$

$$\int_{-\infty}^\infty x^2e^{\left(\frac{-x^2}{2a^2}\right)}dx = a^3\sqrt{2\pi}$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{4a^2}\right)}e^{(-ikx)}dx = 2a\sqrt{\pi}e^{(-k^2a^2)}$$

$$\int_{-\infty}^\infty x^2e^{(-2a^2x^2)}dx = \frac{\sqrt{2\pi}}{8a^3}$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{4a^2}\right)}e^{(-ikx)}dx = 2a\sqrt{\pi}e^{(-k^2a^2)}$$

$$\int_{-\infty}^\infty x^2e^{\left(\frac{-x^2}{2a^2}\right)}dx = a^3\sqrt{2\pi}$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{2a^2}\right)}dx = a\sqrt{2\pi}$$