UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS AND ENGINEERING

SUPPLEMENTARY EXAMINATION 2012/2013

TITLE OF PAPER: QUANTUM MECHANICS

COURSE NUMBER: P342

TIME ALLOWED : THREE HOURS

THERE ARE **FIVE** QUESTIONS IN THIS PAPER. ANSWER ANY **FOUR** QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER HAS EIGHT PAGES INCLUDING THE COVER PAGE.

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Question One

(A) (i) Write down the Lorentz transformation equations relating the coordinates of an event in two different inertial frames of reference moving with relative velocity along the x-axis. (4 marks)

(ii) A particle is moving in the S' frame with velocity components Ux' and Uy'. Show that they are related to the velocity components Ux and Uy measured by an observer in frame S moving with velocity 'v ' relative to S' as follows:

$$U_x' = \frac{U_x - v}{1 - vU_x / c^2}$$

$$U_y' = \frac{U_y\sqrt{1 - v^2 / c^2}}{1 - vU_x / c^2}$$

(6 marks)

(iii) A spaceship moving away from the Earth at a velocity 0.75 c with respect to the Earth launches a rocket in the direction away from the Earth that attains a velocity 0.75 c with respect to the spaceship. What is the velocity of the rocket with respect to the Earth?
What would be your result if solved classically? Comment

(5 marks)

(B) (i) State the principle of simultaneity in relativity. (2 marks)

(ii) The period of a simple pendulum is measured to be 3.0 s in the reference frame of the pendulum. What will be the period measured by an observer moving at 0.95 c relative to the pendulum? (3 marks)

(iii) An electron moves at a speed of 0.25 c. Given that its rest energy is 0.511 MeV, find its total energy and kinetic energy in eV.

(4+1 marks)

Question Two

(A) A free electron has wave function $\Psi(x,t) = \sin(kx - t)$.
Given that the wave number k = 50(nm)$^{-1}$, determine

(i)	de Broglie wavelength,	(2 marks)
(ii)	momentum,	(2 marks)
(iii)	kinetic energy and	(2 marks)
(iv)	speed of the electron.	(2 marks)

(B) (i) State what the terms **expectation value** and **standard deviation** represent.

(2 marks)

(ii) Calculate the position and momentum uncertainties in the wave function

$$\psi(x) = \left(\frac{1}{2\pi a^2}\right)^{1/4} e^{-x^2/4a^2}$$ (14 marks)

(iii) Use the above results to prove Heisenberg uncertainty rule. (1 mark)

Question Three

(A) (i) State any two properties of an acceptable wave function in quantum mechanics. (4 marks)

(ii) Distinguish between phase velocity and group velocity of a wave packet. (2 marks)

(iii) Given that the momentum of a classical particle p = mv, show that the group velocity represents a wave packet (i.e. $v_g = v$). (4 marks)

(B) The wave function of a particle is $\psi(x) = A\, e^{-ax}$ where a >0.

(i) Normalise the above wave function (9 marks)

(ii) Find the interval from the origin such that the probability of finding the particle in this interval is 50% (6 marks)

Question Four

(A) Show that the time independent Schrodinger wave equation for a one dimensional harmonic oscillator can be obtained as

$$\frac{d^2\psi_n}{d\rho^2} = (\rho^2 - \lambda)\psi_n$$

where $\rho = \alpha x$, $k = m\omega^2$, $\rho^2 = \frac{\sqrt{mk}}{\hbar}x^2$ and $\lambda = \frac{2E}{\hbar\omega}$

(6 marks)

(B) By substituting $\psi_n(\rho) = e^{-\rho^2/2}H_n(\rho)$ obtain the Hermite polynomial equation

$$\frac{d^2H_n(\rho)}{d\rho^2} - 2\rho\frac{dH_n(\rho)}{d\upsilon} + (\lambda - 1)H_n(\rho) = 0$$

(8 marks)

(C) Solutions of the above Hermite polynomial equation are the Hermite polynomials $H_n(\rho) = (-1)^n e^{\rho^2}\frac{d^n}{d\rho^n}e^{-\rho^2}$ which exist when $\lambda = 2n + 1$

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(i) Use this definition to determine the solutions $\psi_n(\rho)$ for n = 0, n = 1 and n = 2.

For each solution, determine the corresponding eigen values E_n.

(9 marks)

(ii) Explain why the minimum energy E_0 is not equal to zero. (2 marks)

Question Five

(A) Write down the expressions for the quantum mechanical operators corresponding to the following classical variables:

(i) Position 'x'
(ii) Linear momentum 'p_x'
(iii) Time 't'
(iv) Total energy 'E' (4 marks)

(B) (i) Write down the commutative law for two operators $\hat{A}$ and $\hat{B}$. (2 marks)

(ii) What is the physical significance if two operators do not commute? (2 marks)

(C) (i) State what is meant by an eigenfunction. (2 marks)

(ii) Show that if ψ is an eigenfunction of operator $\hat{A}$, it is an eigenfunction of $\hat{A}^2$. (3 marks)

(iii) The eigenvalue of a certain function $\varphi_{(x)}$ is ħk when operated on by the momentum operator. What is the $\varphi_{(x)}$? (4 marks)

(iv) Given an operator $\hat{H} = \frac{-d}{dx^2} + x^2$, show that $\psi_1 = A_1 e - x^{2/2}$ and $\psi_2 = A_2 xe - x^{2/2}$ are eigenfunctions of $\hat{H}$.

Find their eigenvalues. (8 marks)

Appendix 1

Definite integrals

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2} x^3 dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2} x^5 dx = \frac{1}{a^3}$$

$$\int_0^\infty e^{-ax^2} x dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^2 dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty x^{1/2} e^{-\lambda x} dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty x^4 e^{ax^2} dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x - 1)^2} dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^3}{(e^x - 1)} dx = \frac{\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{(e^x - 1)} dx = \frac{2.61\pi^{1/2}}{2}$$

$$\int_0^\infty e^{-ax} dx = \frac{1}{a}, (a > 0)$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{2a^2}\right)} dx = a\sqrt{2\pi}$$

$$\int_{-\infty}^\infty x^2 e^{\left(\frac{-x^2}{2a^2}\right)} dx = a^3\sqrt{2\pi}$$

$$\int_{-\infty}^\infty e^{\left(\frac{-x^2}{4a^2}\right)} e^{(-ikx)} dx = 2a\sqrt{\pi} e^{\left(-k^2 a^2\right)}$$

Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan –Boltzmann constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}