UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2012 / 2013

TITLE OF PAPER: SOLID STATE PHYSICS.

COURSE NUMBER: P 412

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) Define the terms *lattice, primitive unit cell* and *conventional unit cell* of a crystal. (3 marks)

(b) Draw ***one*** figure showing a conventional unit cell and also the primitive cell of an fcc lattice. (3 marks)

(c) One side of the conventional unit cell of an f.c.c. lattice is 3 Å . What is the volume of its primitive unit cell? (2 marks)

(d) Calculate the separation between two (123) planes of an orthorhombic lattice with cell lengths, a = 0.82 nm, b = 0.94 nm and c = 0.75 nm in the x, y, z directions. (4 marks)

(e) (i) What is meant by packing fraction of a crystal? (2 marks)
(ii) Determine the packing fraction of a bcc crystal. (4 marks)

(f) A first order reflection from the (111) planes of a cubic crystal was observed at a glancing angle of 11.2^0 when x-rays of wavelength 154 pm were used. Calculate the length of the side of each cell. (1 pm = 10^{-12} m). (3 marks)

(g) In the X-ray photograph of a cubic lattice, lines are observed at the following Bragg angles in degrees: 6.6, 9.2, 11.4, 13.1, 14.7. Identify the lattice type. (4 marks)

Question Two

(a) (i) State what is meant by *density of states,* in terms of frequency ω, as applied in lattice dynamics. (2 marks)

(ii) By considering the boundary conditions on lattice waves in a cubic crystal, prove that for a three dimensional system, the density of states is given by the expression

$$D(\omega) = \frac{V\omega^2}{2\pi^2 v^3}$$

where 'V' is the volume, and 'v' is the velocity of sound. (8 marks)

(b) (i) What does Debye approximation mean? (3 marks)

(ii) Show that by applying Debye approximation, the density of states for a system with one atom per unit cell is given by

$$D(\omega) = \frac{9N\omega^2}{\omega_D^{\ 3}}$$

where 'N' is the number of atoms and 'ω_D' is the Debye frequency. (7 marks)

(c) Use the result in (b) above to show that the zero point energy of a lattice is given by

$$E = \frac{9N}{8}\hbar\omega_D$$

[Given: the mean energy of a harmonic oscillator is

$$\bar{\varepsilon} = \hbar\omega\left(\frac{1}{2} + \frac{1}{e^{\hbar\omega/kT} - 1}\right)$$]. (5 marks)

Question Three

(a) State the assumptions Drude made in his ***free electron theory*** of metals. (3 marks)

(b) Define the terms ***mean free path*** and ***mobility*** of an electron. (2+2 marks)

(c) Show that according to the Drude theory, the d.c. electrical conductivity of a metal can be expressed as:

$$\sigma = \frac{ne^2\tau}{m},$$ where the symbols have their usual meanings. (10 marks)

(d) (i) State the ***Wiedemann - Franz*** law. (3marks)

(ii) Write down the expression for the ***Lorenz number*** and calculate its value. (2 + 3 marks)

Question Four

(a) (i) Define 'Fermi energy'. (2 marks)

(ii) Derive an expression for the density of states of a system of electrons, given that the Fermi energy:

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} ,$$ (6 marks)

where the symbols have their usual meanings.

(iii) Calculate the density of energy states at 2.05 eV energy, for a system of electrons in a volume of 1 cm^3. (8 marks)

(b) (i) Show that the electronic contribution to heat capacity of a metal is proportional to absolute temperature. (6 marks)

(ii) Discuss the heat capacity of metals, explaining the difference, if any, between the above theory and the experimental values. (3 marks)

Question Five

(a) Using silicon as an example, explain how the electrical conductivity of a semiconductor can be increased by doping.

(6 marks)

(b) With the help of an appropriate diagram, derive an expression for the effective density of states in the conduction band of a semiconductor. Assume: $(\epsilon-\epsilon_F) \gg kT$.

(10 marks)

[Given: Fermi -Dirac distribution function: $f(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\varepsilon_F)/kT}+1}$]

(c) A doped semiconductor has electron and hole concentrations of 2×10^{13} cm^{-3} and 1.41×10^{13} cm^{-3} respectively. Calculate the electrical conductivity of the sample.

(5 marks)

[Take: $\mu_n = 4200$ cm^2 V^{-1}s^{-1} and $\mu_p = 2000$ cm^2 V^{-1}s^{-1}]

(d) Discuss briefly the process of photoconductivity in semiconductors. (4 marks)

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Plank's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	$1.67 \times 10^{-27\ kg}$
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan- Boltzmann constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of ${}_2^4$ He atom		6.65×10^{-27} kg
Mass of ${}_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ L\ mol^{-1}$