UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2012/2013

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE AND THE LAST PAGE WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27} kg = 931.5 MeV/c^2$

Planck's constant $h = 6.63 \times 10^{-34} Js$

Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$

Avogadro's number $N_A = 6.022 \times 10^{23} mol^{-1}$

Speed of light (vacuum) $c = 3.0 \times 10^8 m/s$

electron mass $m_e = 9.11 \times 10^{-31} kg = 5.4858 \times 10^{-4} u = 0.511 MeV/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27} kg = 1.008665 u = 939.573 MeV/c^2$

proton mass $m_p = 1.6726 \times 10^{-27} kg = 1.0072765 u = 938.280 MeV/c^2$

$1 year = 3.156 \times 10^7 s$

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2 fm$

The table of nuclear properties is provided in the last page.

Question 1: SEMF, Binding Energies and Nuclear Reactions.....................

(a) Using the observation that the nuclear radius $r = r_o A^{1/3}$, estimate the average mass density of a nucleus. (3)

(b) Briefly describe the origin of the various terms in the Semi-Empirical Mass Formula. [Note: detailed mathematical expressions and values of constants are not required]. (5)

(c) Suggest a simple reason why the ${}^{12}_{6}C$ nuclide has a higher binding energy (i.e more stable) than ${}^{12}_{7}N$, even though they are isobars. (2)

(d) Given that the stable sodium isotope is ${}^{23}_{11}Na$, what type of radioactivity would you expect from:

i. ${}^{22}Na$ (3)

ii. ${}^{24}Na$ (3)

(e) Supply the missing particles in the following processes:

i. $\bar{\nu} +^{3} He \longrightarrow$ (2)

ii. $e^{-} +^{8} B \longrightarrow$ (2)

iii. ${}^{40}K \longrightarrow \bar{\nu}+$ (3)

iv. $\nu +^{12} C \longrightarrow$ (2)

Question 2: Rutherford Scattering (Quantum) ....

In the **plum pudding model** of the atom, the atom was assumed to be made up of electrons immersed in a šoup' of positive charge.

(a) Describe the experiment and its' results, used to disprove this model. (4)

(b) In what way did the results of the experiment disprove the model? (4)

(c) Neglecting the coulomb interaction, show, using collision theory, that large angle scattering is not possible when an α-particle collides with an electron, where there electron is initially at rest in the pudding. (10)

(d) The differential cross section for the scattering of a projectile (of mass m, initial velocity $v = |\vec{v}|$, initial momentum $p = |\vec{p}|$ and unit charge) on a much heavier target can be shown to be

$$\frac{d\sigma}{d\Omega} = 4Z^2\alpha^2(\hbar c)^2\frac{p'^2}{vv'q^4},$$

where p' and v' are the final momentum and speed of the projectile, α is the fine structure constant, $\vec{q} = \vec{p} - \vec{p}'$, and other symbols have their usual meanings.

i. Show that $q = 2p\sin(\theta/2)$ for elastic scattering, where θ is the scattering angle. [Hint: Draw a kinematics diagram] (3)

ii. Using non-relativistic approximations for velocities and momenta together with the result above, show that for scattering of very light particles on heavy particles the cross section can be written as (4)

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2(\hbar c)^2}{4E^2\sin^4(\theta/2)},$$

where symbols are as described above.

Question 3: Radioactive Decay ....

(a) For the following γ transitions, give all permitted multipoles and indicate which multipole might be most intense in the emitted radiation.

i. $\frac{9}{2}^- \rightarrow \frac{7}{2}^+$ (1)

ii. $\frac{1}{2}^- \rightarrow \frac{7}{2}^-$ (1)

iii. $1^- \rightarrow 2^+$ (1)

iv. $4^- \rightarrow 2^+$ (1)

v. $\frac{11}{2}^- \rightarrow \frac{3}{2}^+$ (1)

(b) Explain why a transition from 0^+ to 0^+ will not allow any γ radiation. (2)

(c) An Even-Z, even-N nucleus has the following sequence of levels above its 0^+ ground state: (10)
$2^+(89keV)$, $4^+(288keV)$, $6^+(585keV)$, $0^+(1050keV)$, $2^+(1129keV)$.
Draw an energy level diagram and show all reasonably probable γ transitions and their dominant multipole assignments.

(d) Given the following nuclides: ${}^{60}_{27}Co$ and ${}^{32}_{15}P$, show by actual calculation, which of these nuclides will decay by

i. β^+ emission (4)

ii. β^- emission (4)

Question 4: Short Answer Questions ....

(a) Define the following terms:

i. Isotope (1)

ii. Isobar (1)

iii. Isospin (1)

(b) List the four fundamental interactions of nature together with the particles that mediate each type of interaction. (8)

(c) Write short notes on the following processes:

i. Fission (4)

ii. Fusion (4)

(d) Give two examples of each

i. Quarks (2)

ii. Leptons (2)

iii. Gauge Bosons (2)

Question 5: The Yukawa Potential ....

(a) In classical electrodynamics, the scalar field $\phi(\vec{r})$ produced by an electron located at the origin is given by the Poisson equation (10)

$$\nabla^2\phi(\vec{r}) = -4\pi\delta(\vec{r}).$$

Show that the field is given by $\phi(r) = \frac{e}{r}$.

(b) For a nucleon, the scalar field satisfies the Klein-Gordon equation (10)

$$\left(\nabla^2 - \frac{1}{r_0^2}\right)\phi(\vec{r}) = 4\pi g\delta(\vec{r}),$$

where r_0 is the effective range of the interaction described by the field. Show that the radial dependence of the field is given by

$$\phi(r) = -g\frac{e^{-r/r_0}}{r}$$

(c) Show that the range r_0, in the above equation is given by the relation $r_0 = \hbar/mc$ using the fact that the boson, with mass m, is a virtual particle and can therefore exist only for a time Δt given by the Heisenberg uncertainty relation. (5)

NB In spherical coordinates

$$\delta(\vec{r}) \equiv \frac{1}{r^2}\delta(r)\delta(\cos\theta)\delta(\varphi);$$

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi}$$

IDENTITIES

$$\nabla^2 u \equiv \nabla\cdot\nabla u$$

$$\int dx\frac{1}{x_0}\exp(x/x_0) = \ln(x/x_0) + \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(x/x_0)^n$$

Gauss' Divergence theorem

$$\int_V d\vec{r}\nabla\cdot\nabla u(r) = \int_S dS\frac{du(r)}{dr}$$

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%