UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2012/2013

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P 461

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question One

(a) (i) What is meant by ***statistical weight*** of a system of particles? (2 marks)

(ii) What is its significance on the properties of the system? (2 marks)

(b) (i) A system has 6 distinguishable particles arranged in 2 non-degenerate energy levels. What are the possible macrostates? (2 marks)

(ii) Find the number of microstates corresponding to each macrostate and hence determine the most probable configuration. (8 marks)

$$W = N!\prod_{S}\left(\frac{g_s^{\,n_s}}{n_s!}\right)$$

(c) (i) Define 'density of states' in phase space. (2 marks)

(ii) Derive an expression for the volume element in phase space, in terms of energy, and show that the density of states

$$g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon,$$

where the symbols have their usual meanings. (9 marks)

Question Two

(a) Derive the partition function of a classical gas:

$$Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$$ (8 marks)

(b) Show that the pressure of a classical gas is

$$P = NkT\frac{\partial \ln Z}{\partial V}$$

Hence derive the ideal gas equation $PV = NkT$ (10 marks)

(c) Calculate the translational partition function of a hydrogen molecule confined to a volume of 100 cm^3 at 300 K. You may assume that the hydrogen molecules behave like a classical gas. (7 marks)

Question Three

(a) Show that in a system of bosons, for the most probable configuration, the distribution of the bosons can be represented as:

$$n_S = \frac{g_S}{e^{-(\alpha+\beta\varepsilon_S)} - 1},$$

where the symbols have their usual meanings.

Given: Statistical weight of a system of bosons, $W = \prod_S \frac{(g_S - 1 + n_S)!}{(g_S - 1)!n_S!}$

(12 marks)

(b) (i) State what each symbol represents in the Bose-Einstein condensation equation:

$$\frac{N'}{N} = \left(\frac{T}{T_B}\right)^{3/2}.$$

(ii) Find the relationship between the number of particles N_0 in the ground state and the temperature.

(iii) Draw a sketch to show how N_0 varies with temperature.

(6 marks)

(c) In a Bose-Einstein condensation experiment, 10^7 rubidium (atomic mass = 85.47 g/ mol) atoms were cooled down to a temperature of 200 nK. The atoms were confined to a volume of 10^{-15} m^3 .

(i) Calculate the condensation temperature T_B. (4 marks)

(ii) Calculate how many atoms were in the ground state at 200 nK. (3 marks)

given $T_B = \frac{h^2}{2\pi mk}\left(\frac{N}{2.612V}\right)^{2/3}$; Avogadro's number $N_A = 6.02 \times 10^{23}$ /mol.

Question Four

(a) The quantum statistical expression derived by Max Planck for the spectral distribution of energy from a black body is expressed as:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

Derive the spectral distribution under short and long wavelength limits.

(8 marks)

(b) (i) Use the above Planck's distribution function to show that the total energy radiated is proportional to the fourth power of the absolute temperature of the body. (See appendix for definite integrals) (10 marks)

(ii) Given that the proportionality constant in the above expression for total energy is equal to σ (4/c), where σ is the Stefan-Boltzmann constant, and c is the velocity of light, calculate the value of σ.

(7 marks)

Question Five

(a) (i) State what is meant by ***Fermi energy***. (2 marks)

(ii) Find the probabilities that an electron can have energies 0.1 eV and 1.0 eV above the Fermi level at 300 K and 400 K respectively. Comment on the results.

(7 marks)

(b) (i) By deriving an appropriate expression, show that the contribution of electrons towards the heat capacity of a material is proportional to its temperature.

(8 marks)

(ii) In sodium, there are about 2.6 x10^{28} electrons per cubic meter. Calculate the value of the heat capacity per electron of sodium at 300 K. (8 marks)

Given: $N = \dfrac{8\pi V(2m)^{3/2}}{3h^3}\varepsilon_F{}^{3/2}$

Appendix 1

Various definite integrals.

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\, x\, dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2} x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\, x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\, x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\, x^5\, dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\, e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\, dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	$3.00 \times 10^{8}\ ms^{-1}$
Planck's constant	h	6.63×10^{-34} J.s
Boltzmann constant	k	$1.38 \times 10^{-23}\ JK^{-1}$
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	$8.31\ J\ mol^{-1}\ K^{-1}$
Avogadro's number	N_A	$6.02 \times 10^{23}\ mol^{-1}$
Bohr magneton	μ_B	$9.27 \times 10^{-24}\ JT^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ Hm^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\ Wm^{-2}K^{-4}$
Atmospheric pressure		$1.01 \times 10^{5}\ Nm^{-2}$
Mass of $_2^4$ He atom		6.65×10^{-27} kg
Mass of $_2^3$ He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		$22.4\ L\ mol^{-1}$