UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2012/13

TITLE OF PAPER: COMPUTATIONAL METHODS II

COURSE NUMBER: P482

TIME ALLOWED: 3 HOURS

INSTRUCTIONS:

ANSWER ANY FOUR OUT OF SIX QUESTIONS.

CARRY SIX SIGNIFICANT DIGITS IN YOUR CALCULATIONS.

EACH QUESTION CARRIES 25 MARKS.

MARKS FOR DIFFERENT SECTIONS ARE SHOWN ENCLOSED IN SQUARE BRACKETS.

THIS PAPER HAS 6 PAGES INCLUDING THIS PAGE.

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1 (a) Explain the difference between compile-time errors and a run-time errors, give two examples of each. [4]

(b) Identify the errors, **if any**, in the following fragments of C++ code. Assume that all variables are already declared.

(i)
```
if (a ≤ b) {
a = b+z;
b = -1;
}
```
[1]

(ii)
```
for (i =20; i <=10; i++ )
```
[1]

(iii)
```
cin<<x<<y;
r = x**y;
```
[1]

(iv)
```
a = 5;
b = 11;
c = a+b;
cout<<a<<b<<c<<"endl";
```
[1]

(c) Show the value of x after each of the following statements is performed:

(i) x = fmod(11,4) [2]

(ii) x = floor(7.5) [2]

(iii) x = ceil(-7.01) [2]

(iv) x = ceil(pow(fmod(18,8),3) [3]

(d) Translate the following expressions into C++ statements:

(i) $y = e^{\sqrt{x}}$ [2]

(ii) $y = e^{-\frac{x^2}{2}}$ [2]

(iii) $y = (3 - |x|)^{(5-n)}$ [2]

(iv) $y = 1 - \frac{1}{1/_{1-x}}$ [2]

2 (a) Use the Newton-Raphson method to find a root of the *sin* function to a tolerance $\epsilon = 0.00001$, starting with an initial guess of $x = 3$. The working must be clearly shown. [7]

(b) Find a root of the function,

$$f(x) = 2x + \ln(x) - 1$$

using the Newton-Raphson method to a tolerance $\epsilon = 0.00001$. Take the initial guess as $x = 1$. The working must be shown. [8]

(c) Make a model and write an algorithm to simulate the motion of a body sliding down a frictionless incline at an angle θ, using the **ordinary** Euler method. Assume that at time $t_0 = 0$, the body has a velocity v_0 and is at a position s_0 from the top of the incline. You have to generate values of position s and velocity v at N instances of time t between t_0 and t_N. [10]

3 (a) Use the trapezoidal rule to evaluate the integral

$$I = \int_0^1 e^{-x^2} dx,$$

using $n = 6$ intervals. [8]

(b) Integrate the function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{x^2}{2}},$$

using the Simpson rule with $n = 6$ intervals between $x = 0$ and $x = 1$. [10]

(c) Write an algorithm to simulate the discharging of a capacitor from an initial charge Q_0, through a resistor R, as a function of time from $t = t_0$ to $t = t_N$. Use the **ordinary** Euler method with N steps. The capacitor discharge equation is:

$$\frac{dQ}{dt} = -\frac{1}{RC} Q,$$

where C is the capacitance. [7]

4 (a) Apply the **improved** Euler method to the following initial value problem: Consider the ordinary differential equation,

$$\frac{dy}{dt} = x + y \qquad \text{where} \quad y(0) = 0.$$

Using a step size of $\Delta x = h = 0.2$, compute values of y_i for $i = 0$ to 5. Also find the error at each step given that the exact solution for the differential equation above is,

$$y(x) = e^{x} - x - 1.$$

Suggest how the magnitude of the error could be reduced. [10]

(b) A ball at 1200 K is allowed to cool down in air at an ambient temperature of 300 K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by:

$$\frac{d\theta}{dt} = -2.2067 \times 10^{-12}(\theta^4 - 81 \times 10^8),$$

where θ is in K and t in seconds. If $\theta_0 = 1200$ K at $t_0 = 0$ seconds, find the temperature at $t_2 = 480$ seconds using the Runge-Kutta 4th order method. Assume a step size of $\Delta t = h = 240$ seconds and also that at time $t_1 = 240$ seconds the temperature $\theta_1 = 675.65$ K. [15]

5 (a) Write a short C++ program to enter a three-by-four matrix into a two dimensional array. [6]

(b) Solve the following system of equations by the Gaussian elimination method:

$$\begin{aligned} 2x_2 + x_3 &= -8 \\ x_1 - 2x_2 - 3x_3 &= 0 \\ -x_1 + x_2 + 2x_3 &= 3. \end{aligned}$$

The steps of the working must be clearly shown. [10]

(c) Use the Gaussian elimination method to show that the following system of equations has no solutions:

$$\begin{aligned} x_1 - 2x_2 - 6x_3 &= 12 \\ 2x_1 + 4x_2 + 12x_3 &= -17 \\ x_1 - 4x_2 - 12x_3 &= 22. \end{aligned}$$

Show each step of the working. [9]

6 (a) What does the term "Monte Carlo Method" refer to? [3]

(b) Explain the lines numbered (i) to (iv) in the following program that produces random numbers. [8]

```
#include <iostream>
#include <cstdlib> ........................................................ (i)
#include <ctime> .............................................................. (ii)
using namespace std;
int main( )
{
    srand((unsigned)time(0)); ............................................(iii)
    int random_interger;
    for (int index = 0; index < 20; index ++)
    {
      random_integer = (rand( )%10)+1; ...........................(iv)
      cout << random_integer << endl;
    }
  Return 0;
}
```

(c) Describe:

(i) Sample Mean Monte Carlo integration; [3]

(ii) Hit-or-miss Monte Carlo integration. [4]

(d) Define an "object" and explain what Object-oriented programming is. [2]

(e) Use a single sentence each to briefly explain the lines numbered (i) to (v) in the following C++ class definition: [5]

```
class Ratio  ................................................ (i)
{
public: ........................................................(ii)
    void assign(int, int);.......................... (iii)
private: ......................................................(iv)
    int num, den; .................................... (v)
};
```

APPENDIX

Simpson rule

$$\int_a^b f(x)dx = \frac{h}{3}(S_0 + 4S_1 + 2S_2)$$

Ordinary Euler method

$$f(t_{i+1}) = f(t_i) + \frac{df}{dt}\Delta t$$

Improved Euler method

$$\frac{dy}{dx} = f(x,y)$$

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_{i+1}, y_i + k_1)$$

Runge – Kutta 4th Order Method

$$\frac{dy}{dx} = f(x,y)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$$

$$k_4 = f(x_i + h, y_i + k_3 h)$$

END OF P482 EXAMINATION