UNIVERSITY OF SWAZILAND

29

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2013/2014

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS

EACH QUESTION CARRIES 25 MARKS

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE

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QUESTION 1

30

(a) Derive the basic kinematic equation, $x = x_0 + v_0 t + \frac{1}{2}at^2$. **(5 marks)**

(b) A body is projected with a velocity v_0 up an inclined plane that makes an angle ϕ with the horizontal. The angle of projection with the horizontal is θ $(\theta > \phi)$. Find distance d where the body lands along the incline in terms of $g, \theta, \phi,$ and v_0. See the illustration in Figure 1. **(10 marks)**

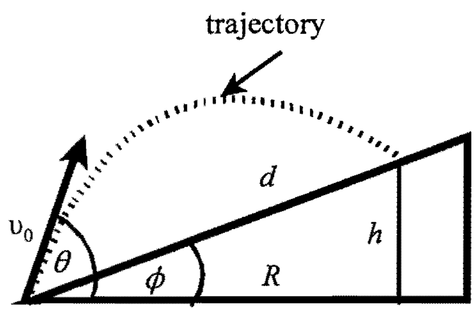

Figure 1.

(c) Derive a generalized expression for the acceleration in Plane Polar Coordinates, given that $\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$ and $\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$ **(10 marks)**

QUESTION 2

3(

(a) Three blocks ($m_1 = 2.00$ kg, $m_2 = 3.00\ kg$, and $m_3 = 4.00$ kg) are arranged on a frictionless table as in Figure 2. A horizontal force $F = 18.0$ N is applied to the mass m_1.

(i) What is the acceleration of the system? **(2 marks)**

(ii) Find the magnitude of the force of contact between m_1 and m_2 and between m_2 and m_3. **(6 marks)**

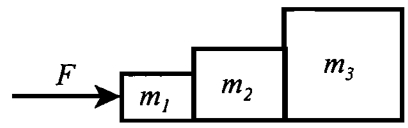

Figure 2.

(b) A smooth (no friction) hemispherical bowl of radius R is fixed with its rim horizontal and uppermost. A particle of mass m moves inside the bowl with a speed v in a horizontal circle. The particle makes an angle θ with the vertical (see Figure 2).

(i) Make a correct resolved force diagram for the particle from which you can make useful equations. **(4 marks)**

(ii) Determine an equation that relates the angle θ, with the velocity v, the radius R, and the gravitational acceleration g **(6 marks)**

(iii) Find the angle θ if $v = \sqrt{\frac{16Rg}{15}}$ **(7 marks)**

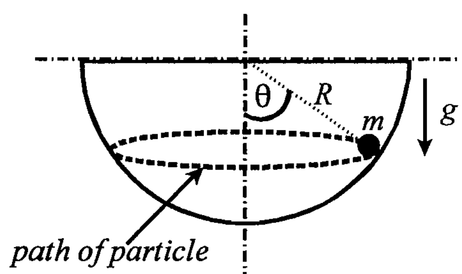

Figure 2

QUESTION 3 32

(a) A uniform hemisphere of mass M, radius R and density ρ is placed on its flat part such that its centre of curvature is at the origin. Starting with the infinitesimal element in spherical coordinates determine by calculation the centre of mass of the hemisphere. **(11 marks)**

(b) A sand spraying locomotive sprays sand horizontally into a freight car. Initially, the freight car is at rest. The engineer in the locomotive maintains the speed so that the distance to the freight car is constant. The sand is transferred to the freight car at a rate $\frac{dm}{dt}$. The sand arrives at the freight car with a horizontal velocity u. Find the speed of the freight car as a function of the initial mass M_0 (mass of freight car before the spraying begins) and final mass M_f, in terms of M_0, M_f and u.
(14 marks)

QUESTION 4

33

(a) A projectile is launched with velocity $v_0$from the earth's surface. Use the work-energy theorem to determine the velocity of the projectile as a function of $G, Me, Re,$ and v_0, where G is the Universal gravitational constant, M_E the mass of the earth and R_E the radius of the earth. **(9 marks)**

(b) A certain uniform spring has a spring constant k. What happens to the spring constant when the spring is cut in half? Explain your answer. **(4 marks)**

(c) A particle is under the potential energy function:

$U(r) = a\left(\frac{b}{2r^2} - \frac{1}{r}\right)$, where a and b are positive contants.

(i) Find the force acting on the particle. **(3 marks)**

(ii) Determine the equilibrium point(s). **(3 marks)**

(iii) Determine the stability of the equilibrium point(s). **(3 marks)**

(iv) Find the angular velocity of small oscillations about the stable equilibrium point. **(3 marks)**

QUESTION 5

(a) Find the moment of inertia of a very thin hoop of mass M and radius R about its centre of symmetry. **(5 marks)**

(b) Starting with the definition of angular momentum derive an expression that relates torque ($\vec{\tau}$) with angular momentum $\vec{L}$. **(6 marks)**

(c) Figure 5 illustrates a conical pendulum. The length of the string is l and it makes an angle of α with the vertical. Find

- (i) the angular momentum about A, **(4 marks)**
- (ii) the angular momentum about B, **(4 marks)**
- (iii) the torque about A, and **(3 marks)**
- (iv) the torque about B. **(3 marks)**

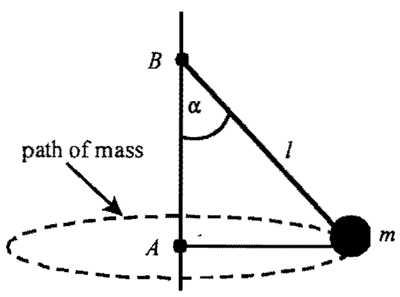

Figure 5.