UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2013/2014

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2\sin\theta drd\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: ELECTROSTATICS 43

(a) The electrostatic force on a charge, due to an ensemble of source charges, can be evaluated by the superposition of the forces between the charge and a single source charge.

i. Describe the principle of superposition of electrostatic forces and write down the mathematical expression that describes superposition. (2)

ii. From the expression for the superposition of the forces, show that the electrostatic field $\vec{E}$ obeys superposition. (2)

iii. Also show that the electrostatic potential V obeys superposition. (2)

iv. Use the electrostatic field $\vec{E}$ to show that electrostatic energy does not obey superposition. (5)

(b) Determine which of the following is a possible electrostatic field (one is not a possible electric field):

i. (4)

$$\vec{E} = k[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}]$$

ii. (4)

$$\vec{E} = k[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}]$$

(c) For the possible electrostatic field above, find the potential using the origin as a reference point. (6)

Question 2: ELECTROSTATIC FIELDS IN MATTER 44

(a) An ideal conductor is a one with an infinite supply of free charges. List the five basic electrostatic properties of an ideal conductor. (5)

(b) An atom placed inside an electrostatic field E acquires a tiny dipole $\vec{p}$. Lets assume a primitive model of the atom, i.e a point nucleus with charge $(+q)$ surrounded by a uniformly charged spherical cloud of total charge $(-q)$ and radius a.

i. Express the dipole moment $\vec{p}$ in terms of E. (1)

ii. Express the magnitude of the dipole moment in terms of the displacement. (1)

iii. Clearly show (stating all assumptions) that the electrostatic field E can be expressed as follows (6)

$$E = \frac{1}{4\pi\epsilon_0}\frac{|\vec{p}|}{a^3}$$

iv. What is the atomic polarizability? (2)

(c) According to Quantum Mechanics, the electron cloud for a hydrogen atom in the ground state (a state endowed with spherical symmetry) has a charge density

$$\rho(r) = \frac{q}{\pi a^3}\exp(-2r/a),$$

where q is the charge of the electron and a is the radius of the cloud.

i. Use Gauss's law to calculate the electrostatic field of the electron cloud. (4)

ii. When the atom is placed inside an external field, the positive charge shifts relative to the negative charge. Assuming the electron cloud is marginally distorted, express the external field in terms of q, a and d (the displacement of the proton relative to the center of the cloud) (4)

iii. Use the result above to determine the atomic polarizability and compare the result with the result of the primitive atom model. (2)

Question 3: Magnetostatics 45

(a) In the discovery of the electron, J.J. Thompson measured the particle's charge-to-mass ratio.

i. Suppose a charged particle enters a region of uniform magnetic field B (into the page). Let the linear extent of the field region be a. If the particle is deflected a distance d from the original line of flight as it travels through the region of the field, show that the particle's momentum is given by (5)

$$|\vec{p}| = qB\frac{a^2 + d^2}{2d},$$

where q is the charge on the particle.

ii. In the first part, Thompson passed the electron beam over mutually perpendicular electric E and magnetic B fields. He then adjusted the electric field till he got zero deflection of the particles. What is the speed of the particles in terms of E and B? (5)

iii. Then he turned off the electric field, and measured the radius of curvature, R. Whats is the charge-to-mass ratio (q/m) of the particles in terms of E, B and R. (5)

(b) Given the definition of the current density $\vec{J} \equiv \frac{d\vec{I}}{da_\perp}$, derive the continuity equation (10)

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}.$$

Question 4: Magnetostatics II... 46

(a) All magnetic phenomena is a result of charges in motion.

i. Write short notes on the magnetization of materials. (4)

ii. Describe each of the following materials: paramagnets, diamagnets and ferromagnets. (6)

(b) The magnetic field due to a current $\vec{I}$ can be determined using either the Biot-Savart law or Ampere's law.

i. Use the Biot-Savart law to show that the magnetic field a distance z above the center of a circular loop of radius R, carrying a steady current I, is given by (5)

$$\vec{B} = \frac{\mu_0 I}{2} \frac{R^2}{(z^2 + R^2)^{3/2}}$$

ii. Use the Biot-Savart law and the result above to find the magnetic field at a point on the axis of long tightly wound solenoid consisting of n-turns per unit length wrapped around a cylindrical tube of radius a carrying a steady current I. (5)

iii. Use Ampére's Law to find the magnetic field at the center of the solenoid described above. (5)

Question 5: Alternating Current Circuits ....

(a) Write short notes on mutual inductance with regard to the operation of a transformer. (5

(b) At some point in time there were people who favored DC distribution systems for electric power while others favored AC distribution systems.

i. Can a DC battery be used as the primary voltage source in a transformer? (1

ii. Explain why it can or cannot be used? (3

(c) A sinusoidal voltage $V(t) = (200V)\sin(\omega t)$ is applied to a series RLC circuit with $L = 10.0\ \ mH$, $C = 100\ \ nF$ and $R = 20.0\ \ \Omega$. Determine the following quantities:

i. the resonant frequency, (2

ii. the amplitude of the current at resonance, (2

iii. the quality factor Q of the circuit, and (2

iv. the amplitude of the voltage across the inductor at the resonant frequency. (2

(d) Suppose at $t = 0$ the capacitor in an LC circuit is fully charged to Q_0. At a later time, $t = T/6$, where T is the period of the LC oscillation, find the ratio of each of the following quantities to their maximum values:

i. charge on the capacitor, (2

ii. energy stored in the capacitor, (2

iii. current in the inductor, and (2

iv. energy in the inductor. (2