UNIVERSITY OF SWAZILAND 48

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2013/2014

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations 49

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z},\ d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z},\ d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi},\ d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: Electrostatics 50

(a) List the three properties of the electrostatic field that make it possible to write the field as a gradient of a potential. (4)

(b) To summarise the main relations in electrostatics, it is necessary to consider the following quantities: The charge density ρ, the electrostatic field E and the electrostatic potential V.

   i. Write down the two relations that connect ρ and V. (2)

   ii. Write down the two relations that connect ρ and $\vec{E}$. (2)

   iii. Write down the two relations that connect V and $\vec{E}$. (2)

(c) The electric potential of some configuration is given by the expression

$$V(\vec{r}) = A\frac{\exp(-\lambda r)}{r},$$

where A and λ are constants.

   i. Find the electric field $\vec{E}$. (5)

   ii. Find the charge density $\rho(r)$. (5)

   iii. Find the total charge Q. (5)

Question 2: Electrostatic Fields in Matter................................5!

(a) Write short notes on polarization in dielectrics by describing the two effects of external electrostatic fields on such materials. (8)

(b) For an ideal conductor, $\vec{E} = 0$ inside.

i. Describe how a conductor mainatains zero field inside when it is placed in an external electrostatic field. (3)

ii. Use $\vec{E}_{ins} = 0$ to show that the charge density $\rho = 0$ inside the conductor. (3)

iii. Use $\vec{E}_{ins} = 0$ to show that a conductor is an equipotential. (3)

iv. Just outside a conductor, the electrostatic field is pependicular. Explain why that is the case. (3)

(c) Prove that if an empty cavity is surrounded by a conductor, $\vec{E} = 0$ inside the regardless of the presence of fields external to the conductor. (5)

Question 3: Magnetostatics ....

(a) Magnetostatics mainly differs from Eletrostatics in that it is concerned with source charges in motion while electrostatatics is concerned with stationary source charges.

i. Describe the electrostatic Gaussian surface associated with Gauss's Law. (1)

ii. Describe the magnetostatic Amperian loop associated to Ampére's Law. (1)

iii. Contrast the Amperian loop with the Gaussian surface. (2)

iv. Qualitatively describe Gauss's law in terms of electric field lines. (3)

v. Qualitatively describe Ampére's law in terms of the current density. (3)

(b) Show that the trajectory of a charged particle particle in a uniform magnetic field is circular. Assume the velocity of the particle is perpendicular to the field. (10)

(c) Suppose that a magnetic field in some region is given by $\vec{B} = kz\hat{x}$, where k is a constant. What is the force on a square loop lying in the yz-plane and centered at the origin. Let the loop be of side a, and carry a current I flowing counterclockwise. (5)

Question 4: Magnetostatcs II 53

(a) Magnetic fields, like electrostatic fields, suffer a discontinuity at a boundary.

i. Draw a pillbox straddling a surface whose Current density is $\vec{K}$. (1)

ii. Show that the integral equivalent of $\nabla \cdot \vec{B} = 0$ is (5)

$$\oint \vec{B} \cdot d\vec{a} = 0.$$

iii. Use $\oint \vec{B} \cdot d\vec{a} = 0$ to show that the perpendicular component of the field is continuous at the surface. (3)

iv. Using an amperian loop on the pillbox running perpendicular to the current show that the tangential component of the field is discontinuous, i.e show that $B^{tant}_{above} - B^{tant}_{below} = \mu_0 K$, where B^{tant} is the tangential component of the field. (6)

v. Hence, deduce that the magnetic field above the surface is related to the magnetic field below the surface by (2)

$$\vec{B}_{above} - \vec{B}_{below} = \mu_0(\vec{K} \times \hat{n}),$$

where $\hat{n}$ is the unit vector normal to the surface.

vi. Compare the discontinuity of the magnetic field at a surface to the discontinuity of electric field at a surface. (2)

(b) Prove that for a closed loop in a uniform magnetic field, in which a steady current flows, the net force is zero. (6)

Question 5: Electrodynamics . . . . [?]

(a) Ohm's law is a rule that is applicable to a lot of materials. Describe how the law holds given that the free charges experience a net force $\vec{F} = q\vec{E}$. Begin by describing the effect of the net force and mention the important equations. (6)

(b) The current in a simple circuit is the same everywhere yet there is a single emf source. Moreover, the drift velocity of the free charges is small yet as soon as a switch is closed all parts of circuits are almost instantaneously affected. Discuss the 'communication' mechanism in such a circuit. (6)

(c) A capacitor C, which has been charged up to potential V_0, is connected to a resistor at time $t = 0$.

i. Determine the charge on the capacitor as a function of time, $Q(t)$. (4)

ii. Determine the current through the resistor as a function of time, $I(t)$. (3)

iii. Calculate the original energy stored in the capacitor. (3)

iv. Integrate $P = VI$ and confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor. (3)