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FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF PHYSICS
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SUPPLEMENTARY EXAMINATION 2013/2014
TITLE OF PAPER : THERMODYNAMICS
COURSE NUMBER: P242/EE202
TIME ALLOWED : THREE HOURS

INSTRUCTIONS : ANSWER ANY FOUR OUT OF FIVE QUESTIONS
EACH QUESTION CARRIES 25 MARKS
MARKS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.
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## INFORMATION

For a monatomic gas: $\gamma=5 / 3$ and $C_{V}=3 R / 2 ; C_{P}=5 R / 2$

For a diatomic gas: $\gamma=7 / 5$
Universal gas constant, $\mathrm{R} \quad=8.31 \mathrm{~mol}^{-1} \mathrm{~K}^{-1}$
Specific heat of water, $\mathrm{c}_{\mathrm{w}} \quad=4190 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$
Latent heat of vaporisation of water, $\mathrm{L}_{\mathrm{V}} \quad=2.256 \times 10^{6} \mathrm{~J} \mathrm{~kg}^{-1}$
Stefan-Boltzmann constant, $\sigma$
$=5.67 \times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$

## OUESTION 1

(a) Show that the pressure of an ideal gas is related to the density of the gas and the mean square speed of the gas molecules as indicated in the equation below.

$$
\begin{equation*}
p=\frac{1}{3} \rho v^{2} \tag{15marks}
\end{equation*}
$$

(b) Assume that the speed of sound in an ideal gas is the same as the root-mean-square speed of the gas molecules and show that the speed of sound in an ideal gas depends on temperature.

> (5 marks)
(c) Consider an ideal monatomic gas whose molecular mass and root-mean-square molecular speed are $4 \times 10^{-3} \mathrm{kgmol}^{-1}$ and $1360 \mathrm{~ms}^{-1}$ respectively.

Find the temperature of the gas.
(a) The change in internal energy of one mole of an ideal gas undergoing a reversible adiabatic process is given by the following equation:
$\mathrm{dU}=-\mathrm{dW}=-\mathrm{pdV}=\mathrm{C}_{\mathrm{V}} \mathrm{dT}$,
where the symbols have their usual meanings.
Show that

$$
\frac{T_{1}}{T_{2}}=\left(\frac{Y_{2}}{Y_{1}}\right)^{\gamma-1}
$$

where 1 and 2 represent the initial and final states.
(b) Two thousand moles of a monatomic ideal gas is taken through the following cyclic process (1) An isobaric expansion from $2 \mathrm{~m}^{3}$ to $4.6 \mathrm{~m}^{3}$ at a pressure of $4 \times 10^{6} \mathrm{~Pa}$; (2) An isochoric decrease in pressure from $4 \times 10^{6} \mathrm{~Pa}$ to $1 \times 10^{6} \mathrm{~Pa}$; (3) An adiabatic compression back to the initial state.
(i) Show the p-V diagram for this cyclic process; (2 marks)
(ii) What is the work done during each step of the cycle and the net work done for the cycle?
(9 marks)
(iii) What is the heat exchanged during each step of the cycle and the net heat for the cycle?
(7 marks)

## OUESTION 3

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(a) Consider an ideal gas which is caused to move through a p-V cycle $A B C A$. A to $B$ is an isochoric process, B to C is an isothermal expansion, and C to A is an isobaric process.
(i) For each leg of the cycle, taking the gas as the system, determine whether each of the following quantities is positive, negative or zero: change in temperature, heat absorbed, work done by gas, change in internal energy, change in entropy.
(10 marks)
(ii) For the whole cycle - A to B to C to A - answer the same questions.
(iii)Operated as stated, is this the cycle of a heat engine or refrigerator? Give reasons for your answer.
(b) A heat reservoir at 373 K is used to evaporate 0.2 kg of water originally at 313 K .
(i)How much energy must flow form the reservoir to do this? (6 marks)
(ii) What is the change entropy of the water?

## QUESTION 4

(a) The overall volume expansion ratio of a Carnot cycle shown in Fig. 1 is 15. The temperature limits of the cycle are $21^{\circ} \mathrm{C}$ (step C -D) and $260^{\circ} \mathrm{C}$ (step A-B)


Fig. 1

Determine the volume ratios of the isothermal and adiabatic processes.
(b) Imagine a Carnot engine which takes 5000 J heat each cycle from the hightemperature reservoir at 300 K and gives out 3500 J to the low-temperature reservoir.
(i) Calculate the temperature of the low-temperature reservoir.
(4 marks)
(ii) What is the thermal efficiency of the cycle?
(c) A real engine takes in 210 kJ of heat at 700 K and expels 140 kJ at 400 K .
(i) If the real engine is replaced by a Carnot engine working between the same two temperatures with an input of 20 kJ , how much heat would be expelled and how much work would it do?
( 5 marks)
(ii) What would be the co-efficient of performance of the Carnot engine if it were used as a heat pump between the same two temperatures?
(4 marks)

## QUESTION 5

(a) A heating element is embedded in a ceramic rod 0.05 m in length and its emissivity is 0.50 . If the ceramic rod is to be maintained at $737^{\circ} \mathrm{C}$ with a power of 1 kW supplied to the element, what should the radius of the rod be?
[Assume that the surroundings are at 0 K and that negligible heat losses occur from the ends of the rod].
(5 marks)
(b) A house without an insulation in the ceiling loses 200 J of heat per second per square metre of the ceiling when the inside temperature is heated to $40^{\circ} \mathrm{C}$ and the outside temperature is $0^{\circ} \mathrm{C}$. The owner decides to add glass wool insulation [thermal conductivity $0.042 \mathrm{~W} / \mathrm{mK}$ ] to the ceiling. How thick should the insulation be in order to cut the heat loss by $80 \%$ ?
(6 marks)
(c) A sphere of radius R , held at temperature To, radiates a s a black body. Determine an expression for the power flux at a distance $d \gg R$ from the centre of the sphere.
(5 marks)
(d) The surface temperature of a spherical mass of molten metal (the source) is $\mathrm{T}_{1}$ and its radius is $r_{1}$. It is surrounded by a spherical shell of inside radius $r_{1}$, outside radius $r_{2}$ and thermal conductivity k . The outside of the shell is at temperature $\mathrm{T}_{2}$. Assuming k is constant, show that the radial rate of flow of heat in a substance between the two concentric spheres is given by

$$
\begin{equation*}
\frac{d Q}{d t}=\frac{4 \pi r_{1} r_{2}\left(T_{1}-T_{2}\right)}{\left(r_{2}-r_{1}\right)} \tag{9marks}
\end{equation*}
$$

