77

UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2013/2014

TITLE OF THE PAPER: COMPUTATIONAL METHODS-I

COURSE NUMBER: P262

TIME ALLOWED:
SECTION A: ONE HOUR
SECTION B: TWO HOURS

INSTRUCTIONS:
THE ARE TWO SECTIONS IN THIS PAPER:

- **SECTION A:** IS A WRITTEN PART. ANSWER THIS SECTION ON THE ANSWER BOOK. IT CARRIES A TOTAL OF **40** MARKS.
- **SECTION B:** IS A PRACTICAL PART WHICH YOU WILL WORK ON A PC AND SUBMIT THE PRINTED OUTPUT. IT CARRIES A TOTAL OF **60** MARKS.

Answer **all** the questions from Section A and **all the questions** from Section B. Marks for different sections of each Question are shown in the right hand margin.

THE PAPER HAS 5 PAGES, INCLUDING THIS PAGE.

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Section A – Use a pen and paper to answer these questions

Question 1

(a) Translate the following expressing into Maple statements.

(i) $sin(\pi/3)$ up to 10 decimal points

(ii) $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+........+\frac{1}{10000}$

(iii) $\int_a^b(\exp(-x^2/2)+x^4/4!)dx$

(iv) $\frac{d^5}{dx^5}(\ln xy)$

[8 marks]

(b) Explain the difference between the following Maple statements:

(i) $a := 3*x+2*I$: and $a := 3*x+2*I$;

(ii) **dsolve**(x^5/5 − x^4/5 − 7 ∗ x^3 + 10); and **dsolve**(x^5/5 − x^4/5 − 7.0 ∗ x^3 + 10);

[4 marks]

(c) Determine the output of the following Maple commands.

(i) > sum(n, n=1..10);

(ii) > [seq(2 ∗ n ∗ (n + 1), n = 1..5)];

(iii) > d := (x, y, z) → $sqrt$(x^2 + y^2 + z^2) : d(0, 1, 1);

(iv) > (1 + I)^2;

[4 marks]

(d) Ten gas particles have speeds of 5.00, 8.00, 12.0, 12.0, 12.0, 14.0, 14.0, 17.0, 18.0 and 20.8 m/s. Write a Maple program that compute the average speed of the particles and the root-mean-square of the speed of the particles.

[4 marks]

Question 2

(a) When using Maple to find the roots of the equation

$$x^5 + x^4 - x = 0,$$

you obtain the following response

$> a := solve(x\hat{}4 + x\hat{}3 - x, x);$

$$a := 0, RootOf(\_Z^4 + \_Z^2 - 1, index = 1), RootOf(\_Z^4 + \_Z^2 - 1, index = 2),$$
$$RootOf(\_Z + \_Z - 1, index = 3), RootOf(\_Z + \_Z - 1, index = 4)$$

(i) What is the meaning of the above response?

(ii) How do you extract all the roots?

(iii) If we use **fsolve**, Maple displays only the real roots, how can you determine the other complex roots?

[6 marks]

(b) In an RLC circuit the frequency f is given by the equation

$$f = \sqrt{\frac{1}{LC} - \frac{R^2}{4C^2}}$$

For $L = 1.5 \times 10^{-14}H$ and $R = 3k\Omega$,

(i) Write a Maple code that calculates C for $f = 100Hz$ numerically in Maple, using the **fsolve** command.

[4 marks]

(ii) Now compute C for a range of frequencies, $f = 100Hz$ to $f = 1kHz$ in steps of $100Hz$. *NB: The above information means f can be generated as a list, with the i^{th} term $f[i] = 100 * i$ and each value of $f[i]$ will correspond to a capacitance $C[i]$*

[6 marks]

(c) The Hamiltonian matrix for a quantum system is given as

$$\begin{bmatrix} -2B & 0 & 0 & J \\ 0 & 0 & J & 0 \\ 0 & J & 0 & 0 \\ J & 0 & 0 & 2B. \end{bmatrix}$$

Write a Maple program the compute to determinant of the matrix.

[4 marks]

Section B – Practical Part

Question 3

The electrostatic potential $V(x)$ inside a cylinder was measured and the following results were obtained :

$$\begin{aligned} V(0.0) &= 52.640,\ V(0.2) = 48.292,\ V(0.4) = 38.270, \\ V(0.6) &= 25.844,\ V(0.8) = 12.648,\ and\, V(1.0) = 0.0, \end{aligned} \tag{1}$$

where x is the distance from the center of the cylinder in metres and V is given in Volts.

(a) Plot the a graph of $V(x)$ versus x using the given data set.

[5 marks]

Symmetry arguments requires that $V(x)$ be an even function of x. Given that

$$V(x) = \sum_{n=0}^{5} a_{2n}x^{2n} = a_0 + a_2x^2 + a_4x^4 + a_6x^6 + a_8x^8 + a_{10}x^{10},$$

(b) Use the LeastSquare procedure in Maple to estimate the values of a_{2n} from the information given in Eq. (1).

[5 marks]

(c) At what distance between $x = 0.0$ and $x = 1.0\,m$ is the electric potential equal to 20 Volts? *You may need to find the roots of the equation*

$$V(x) - 20 = 0.$$

[5 marks]

Question 4

The dynamics of a charged particle in a magnetic field is described by Newton's second law:

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{v} \times \mathbf{B} - \frac{\gamma}{m}\mathbf{v}$$

where $\mathbf{B}$ is the magnetic field, γ represent the coefficient of a damping force, m and q corresponds to the mass and the charge of the particle, respectively.

(a) **Uniform field**. When $\mathbf{B}$ is in the z-direction, then the dynamics are given by four equation:

$$\frac{d}{dt}x(t) = v_x(t), \quad \frac{d}{dt}v_x(t) = b_0 v_y(t) - b_1 v_x(t) \tag{2}$$

$$\frac{d}{dt}y(t) = v_y(t), \quad \frac{d}{dt}v_y(t) = -b_0 v_x(t) - b_1 v_y(t) \tag{3}$$

(i) Write a program to simulate the dynamics of the charged particle. Assume that the initial velocity of the particle is $\mathbf{v}(t=0) = [vx(0), vy(0)] = (1, 0)$ and the initial position $\mathbf{r}(t=0) = [x(0), y(0)] = (0, 0)$. You may choose the Euler method in Maple to find the solutions with the *stepsize* =0.001. $b_0 = 2.0$, and $b_1 = 0$. Plot the trajectory of the particle [x(t),y(t)] for t =0..10. What is the shape of the trajectory of the particle?

[15 marks]

(ii) Choose an appropriate value(s) of b_1 to investigate the effects of the damping force. Plot the trajectory of the particle under the influence of a damping force with the same initial conditions as (i). Discuss your observations.

[10 marks]

(b) **Crossed Electric-Magnetic fields**. Now suppose an electric field $\mathbf{E} = E_0\hat{\mathbf{x}}$ is introduced into the system . In this case we need to add an extra term in the equation describing the change in x-component of the velocity :

$$\frac{dv_x(t)}{dt} = b_0 v_y(t) - b_1 v_x(t) + E,$$

whilst the other equations remain the same. Assume that $E = 2.0$, $b_0 = 2.0$, and $b_1 = 0$, in dimensionless units. Show that the particle released with an initial velocity $\mathbf{v}(t) = 0$ from the origin will move like a leaping frog along the y-axis. Explain why this is so.

[20 marks]