UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2013 /2014

TITLE OF PAPER: SOLID STATE PHYSICS

COURSE NUMBER: P 412

TIME ALLOWED : THREE HOURS

ANSWER ANY **FOUR** OF THE FIVE QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

Question One

(a)

    (i)   Distinguish between a primitive unit cell and a conventional unit cell.

                                                  (2 marks)

    (ii)   Draw the Wigner - Seitz cell of a two-dimensional direct lattice.
State whether it is a primitive or a conventional cell.   (2+1 marks)

    (iii)   Sodium has a density of 0.968 g/cm^3 and a unit cell side of length 4.29 Å.
Determine the lattice type. [Atomic mass of sodium = 22.99 g/mol]

                                                  (5 marks)

(b)   (i)   In the diagram of a cubic unit cell, show a (121) and a (212) plane.

                                                  (2+2 marks)

    (ii)   Calculate the distance between two such planes if the lattice constant is 1 Å.

                                                  (3 marks)

    (iii)   A cubic crystal plane has intercepts 3a, 2a and 1a along x , y, and z axes
where 'a' is the lattice constant. Find the Miller indices of this plane.

                                                  (2 marks)

(c)   (i)   What is meant by 'pcking fraction' of a crystal?   (2 marks)

    (ii)   Determine the packing fraction of a diamond lattice.   (4 marks)

Question Two

(a)   (i)   What is van der Waals-London attractive interaction in inert gas crystals?

(3 marks)

(ii)   Explain how Pauli's exclusion principle is responsible for the repulsive interaction in inert gas crystals.   (3 marks)

(b)   Derive an expression for the total lattice energy of a one -dimensional crystal consisting of a line of 2N ions of alternating charge $\pm$ q at their equilibrium separation 'R_0'.The repulsive interaction may be assumed to be of the form $\lambda\exp(-r/\rho)$.

(12 marks)

(c)   In an ionic crystal, a line of 2N ions of alternating charges (+/-) q have a repulsive potential energy of the form A/R^n between nearest neighbours. Show that at equilibrium separation the potential energy,

$$U_{tot} = \frac{-2Nq^2\ln 2}{R_0}\left(1 - \frac{1}{n}\right).$$

[Given: Madelung constant = 2 ln2]   (7 marks)

Question Three

(a) State **Bragg's law** of crystal diffraction. (2 marks)

(b) X-rays and electrons can both be used for crystal diffraction experiments. If the lattice constant of a crystal is 1 Å, calculate

    (i) the energy of x-ray photons that can be used in a diffraction experiment;

(4 marks)

    (ii) the energy of an electron that can be diffracted by the crystal;

[Given: photon energy $E = hc/\lambda$, electron energy $E = h^2/(2m\,\lambda^2)$]

(4 marks)

    (iii) State why an electron beam cannot be used for the study of bulk materials, as compared to x-rays.

(2 marks)

(c) The geometric structure factor of a crystal is given below.

$$S_G = \sum_{j=1}^{s} f_j \exp[-i2\pi(n_1 h + n_2 k + n_3 l)]$$, where 's' is the number of atoms in the basis and

n_1, n_2, n_3 are fractional coordinates. 'f' is the atomic form factor.

Show that diffraction can occur from planes in a diamond crystal only if

$$(h + k + l) = 4n$$

where 'n' is any integer and h, k, l are the Miller indices. (8 marks)

(d) With the help of a diagram, explain why a bcc lattice does not contain diffraction lines corresponding to (100) planes.

(5 marks)

Question Four

(a)   (i)   Define *Fermi Energy.*   (2 marks)

(ii)   Write down the Fermi Dirac (F-D) distribution function for a system of fermions.
                                                                        (2 marks)

(iii)   Compute the values of the F-D distribution function for the following cases at absolute zero.

1. Energy of the fermion ϵ > Fermi energy ϵ_F
2. Energy of the fermion ϵ < Fermi energy ϵ_F (4 marks)

(iv)   In a single sketch, show how the Fermi function varies with energy at T = 0K and also at T > 0K and comment on the physical meaning of your observations.
                                                                        (5 marks)

(b)   (i)   Using the Schrödinger wave equation, show that the energy of a free electron is:

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \text{ where the symbols have their usual meanings.}$$ (4 marks)

(ii)   Use the results in (b)(i) above to show how the Fermi energy is related to the electron concentration, and hence derive an expression for the density of states of the electrons in a metal.
                                                                        (8 marks)

Question Five

(a)   Explain how electrical conductivity of a pure semiconductor can be increased by:

   (i)   Thermal generation of carriers
   (ii)   Doping.

   Give examples where necessary   (3+3 marks)

(b)   With the help of an appropriate energy band diagram, show that the density of electrons in the conduction band of a semiconductor is given by the expression:

$$N_e = 2\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \exp\left[\frac{\varepsilon_F - \varepsilon_g}{kT}\right]$$

where the symbols have their usual meanings. [Assume $(\epsilon - \epsilon_{F)} \gg kT$]

Given: $$\int_0^\infty \exp(-nx)x^{1/2}dx = \frac{1}{2n}\sqrt{\pi}$$

                                                                    (12 marks)

(c)   A silicon sample is doped with 10^{17}cm^{-3} arsenic atoms. All dopants are ionised.

   1. What is the equilibrium hole concentration?
   2. Where is the Fermi level relative to the centre of the band gap?

   [Intrinsic carrier concentration of silicon is 1.5×10^{10} cm^{-3}]

                                                                    (7 marks)

Appendix 1

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Various definite integrals.

$$\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\, x\, dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\, dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\, x^2\, dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\, x^4\, dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}.$$

$$\int_0^\infty e^{-ax^2}\, x^5\, dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\, dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\, e^{-\lambda x}\, dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\, dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\, dx = \frac{2.61\pi^{1/2}}{2}$$

Appendix 2

Physical Constants.

Quantity	symbol	value
Speed of light	c	$3.00 \times 10^8 \text{ ms}^{-1}$
Plank's constant	h	$6.63 \times 10^{-34} \text{ J.s}$
Boltzmann constant	k	$1.38 \times 10^{-23} \text{ JK}^{-1}$
Electronic charge	e	$1.61 \times 10^{-19} \text{ C}$
Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Mass of proton	m_p	$1.67 \times 10^{-27} \text{ kg}$
Gas constant	R	$8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	$9.27 \times 10^{-24} \text{ JT}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ Hm}^{-1}$
Stefan- Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$
Atmospheric pressure		$1.01 \times 10^5 \text{ Nm}^{-2}$
Mass of $_2^4$ He atom		$6.65 \times 10^{-27} \text{ kg}$
Mass of $_2^3$ He atom		$5.11 \times 10^{-27} \text{ kg}$
Volume of an ideal gas at STP		22.4 L mol^{-1}