UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION: 2013/2014

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27} kg = 931.5 MeV/c^2$

Planck's constant $h = 6.63 \times 10^{-34} Js$

Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$

Avogadro's number $N_A = 6.022 \times 10^{23} mol^{-1}$

Speed of light (vacuum) $c = 3.0 \times 10^8 m/s$

electron mass $m_e = 9.11 \times 10^{-31} kg = 5.4858 \times 10^{-4} u = 0.511 MeV/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27} kg = 1.008665u = 939.573 MeV/c^2$

proton mass $m_p = 1.6726 \times 10^{-27} kg = 1.0072765u = 938.280 MeV/c^2$

$1 year = 3.156 \times 10^7 s$

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2 fm$

Elementary charge, $e = 1.6021 \times 10^{-19}$ C

Electric constant, $\epsilon_0 = 8.854 \times 10^{-12} \quad C^2/J \cdot m$

The table of nuclear properties is provided in the last page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1: Fundamental Forces 198

(a) Briefly discuss each of the four fundamental forces in nature. (16)

(b) In lecture we discussed three unifying theories. Write brief notes on each. (9)

Question 2: Applications of Nuclear Physics ١٩٩

(a) Qualitatively discuss the fission mechanism. (6)

(b) Given that the probability of tunneling through the fusion barrier is given by $P \approx \exp\left(-2\int_a^b \sqrt{\frac{2m[V(r)-E]}{\hbar^2}}dr\right)$, where m is the reduced mass of the two fusing nuclei, b is the classical turning point.

i. Show that $b = \frac{Z_1 Z 2e^2}{4\pi\epsilon_0 E}$ (4)

ii. Evaluate the constants and show that b can also be written as $b = 1430\frac{Z_1 Z_2}{E}\text{fm} \cdot$ keV After evaluating the integral, the probability is found to be $P \approx \exp\left(-\sqrt{\frac{E_B}{E}}\right)$, where $E_B = \frac{2\pi^2 Z_1^2 Z_2^2}{(4\pi\epsilon_0\hbar)^2}m$ (3)

iii. After evaluating the integral, the probability is found to be $P \approx \exp\left(-\sqrt{\frac{E_B}{E}}\right)$, where $E_B = \frac{2\pi^2 Z_1^2 Z_2^2}{(4\pi\epsilon_0\hbar)^2}m$. By evaluating the constants, show that the barrier height is given by $E_B = 1052 Z_1^2 Z_2^2 \frac{mc^2 keV}{GeV}$. (3)

iv. Show that for $E = 1keV$ and $E = 10keV$, the ratio of the probabilities is $\frac{P(10keV)}{P(1keV)} = 10^{10}$ (4)

(c) Discuss the criteria for the performance of a fusion reactor. (5)

Question 3: Bound States for the Inter-nucleon Potential........................ 20

In a lecture we highlighted that the internucleon potential's strength is large enough to accommodate only one bound state. We can show that for a square well in $3-D$ there is a condition that determines if the strength is sufficient to hold a bound state. The following exercises are the steps in deriving that condition.

(a) Using the complete normalized wave function $\psi(r) = \frac{u(r)}{r}$ for the square well with (8)

$$u(r) = A\sin(k_1 r), \qquad for \quad r < R$$

$$u(r) = C\exp(-k_2 r), \qquad for \quad r > R$$

where $k_1 = \sqrt{2m(E+V_0)/\hbar^2}$ and $k_2 = \sqrt{-2mE/\hbar^2}$, show that the expectation value of the potential energy is

$$\langle V \rangle = \int \psi^* V \psi d^3r = -V_0 A^2 \left[R/2 - \frac{1}{4k_1}\sin(2k_1 R) \right]$$

(b) Show that the expectation value of the kinetic energy is (8)

$$\langle T \rangle = \frac{\hbar^2}{2m} \int |\frac{\partial \psi}{\partial r}|^2 d^3r = \frac{\hbar^2}{2m} A^2 \left[k_1^2 R/2 + k_1/4\sin(2k_1 R) + k_2/2\sin^2(k_1 R) \right]$$

(c) Show that for a bound state to exist to exist, it must be true that $\langle T \rangle < -\langle V \rangle$ (5)

(d) Use the results above to determine an inequality for the minimum value of V_0 that can support a bound state. (4)

Question 4: Nuclear Reactions and Decays 201

(a) Consider the reaction: $A(a,b)B$

i. What is meant by the Q-value? (2)

ii. Show that the Q-value is given by the mathematical relation $Q = (M_A + M_a - M_B - M_b)\, c^2$. (3)

iii. Furthermore, derive an expression for Q in terms of the mass excess (4)

(b) Calculate the mass excess for the following nuclides

i. ^4He (3)

ii. ^{60}Fe (3)

iii. ^{203}Tl (3)

(c) Given that the atomic mass of ^{238}U is $238.0508u$ and that the highest energy of an α particle emitted in the decay of ^{238}U to ^{234}Th is 4.196 MeV, calculate the mass of ^{234}Th. (7)

Question 5: The Valley of Stability 202

For this problem consider the empirical mass formula for the internal energy per nucleon

$$e = -a_{vol} + a_{surf}\frac{1}{A^{1/3}} + a_C\frac{Z^2}{A^{4/3}} + a_{sym}\left(\frac{N-Z}{A}\right)^2$$

(a) Describe each of the five terms in the empirical mass formula. (10)

(b) Show that for nuclei on the stability line we have $Z_0 = \frac{A}{2+xA^{2/3}}$, where $x = \frac{a_C}{2a_{sym}}$. (8)

(c) Use the result above to show that for nuclei on the stability line the internal energy per nucleon is given by (7)

$$e = -a_{vol} + a_{surf}\frac{1}{A^{1/3}} + a_C\frac{Z_0}{2A^{1/3}}$$