UNIVERSITY OF SWAZILAND

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FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2013/2014

TITLE OF PAPER: NUCLEAR PHYSICS

COURSE NUMBER: P442

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT TWO PAGES WHEN NECESSARY.

THIS PAPER HAS 8 PAGES, INCLUDING THIS PAGE.

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Useful Data:

1 unified mass unit $(u) = 1.6605 \times 10^{-27} kg = 931.5 MeV/c^2$

Planck's constant $h = 6.63 \times 10^{-34} Js$

Boltzmann's constant $k = 1.38 \times 10^{-23} J/K$

Avogadro's number $N_A = 6.022 \times 10^{23} mol^{-1}$

Speed of light (vacuum) $c = 3.0 \times 10^8 m/s$

electron mass $m_e = 9.11 \times 10^{-31} kg = 5.4858 \times 10^{-4} u = 0.511 MeV/c^2$

neutron mass $m_n = 1.6749 \times 10^{-27} kg = 1.008665 u = 939.573 MeV/c^2$

proton mass $m_p = 1.6726 \times 10^{-27} kg = 1.0072765 u = 938.280 MeV/c^2$

$1 year = 3.156 \times 10^7 s$

nuclear radius, $R \approx r_0 A^{1/3}$, where $r_0 = 1.2 fm$

Elementary charge, $e = 1.6021 \times 10^{-19}$ C

Electric constant, $\epsilon_0 = 8.854 \times 10^{-12} \quad C^2/J \cdot m$

The table of nuclear properties is provided in the last page.

Nuclide	Z	A	Atomic mass (u)	I^π	Abundance or Half life
H	1	1	1.007825	$1/2^+$	99.985%
He	2	4	4.002603	0^+	99.99986%
Li	3	7	7.016003	$3/2^-$	92.5%
Be	4	11	11.021658	$1/2^+$	13.8 s (β^-)
B	5	11	11.009305	$3/2^-$	80.2%
C	6	12	12.00000	0^+	99.89%
N	7	15	15.00109	$1/2^-$	0.366%
N	7	18	18.014081	1^-	0.63 s
O	8	15	15.003065	$1/2^-$	122 s
O	8	16	15.994915	0^+	99.76%
O	8	18	17.999160	0^+	0.204%
F	9	18	18.000937	1^+	110.0 min
Ne	10	20	19.992436	0^+	90.51%
Ne	10	22	21.991383	0^+	9.33%
Na	11	22	21.994434	3^+	2.60 yrs
Mg	12	21	21.000574	0^+	3.86 s
Al	13	27	26.981539	$5/2^+$	100.0%
Si	14	30	29.973770	0^+	3.10%
Si	14	32	31.974148	0^+	105 yrs
P	15	30	29.978307	1^+	2.50 min
P	15	32	31.971725	1^+	14.3 days
S	16	32	31.972071	0^+	95.02%
Cl	17	37	36.965903	$3/2^+$	24.23%
Ar	18	37	36.966776	$3/2^+$	35.0 days
K	19	37	36.973377	$3/2^-$	1.23 s
Ca	20	43	42.958766	$7/2^-$	0.135%
Ca	20	47	46.954543	$7/2^-$	4.54 days (β^-)
Sc	21	47	46.952409	$7/2^-$	3.35 days (β^-)
Fe	26	56	55.934439	0^+	91.8%
Fe	26	60	59.934078	0^+	1.5 Myrs
Co	27	60	59.933820	5^+	5.27 yrs
Ni	28	60	59.930788	0^+	26.1%
Ni	28	64	63.927968	0^+	0.91%
Ni	28	65	64.930086	$5/2^-$	2.52 hrs (β^-)
Cu	29	63	62.929599	$3/2^-$	69.2%
Cu	29	64	63.929800	1^+	12.7 hrs
Cu	29	65	64.927793	$3/2^+$	30.8%
Zn	30	64	63.929145	0^+	48.6%
Ru	44	104	103.905424	0^+	18.7%
Ru	44	105	104.907744	$3/2^+$	4.44 hrs (β^-)
Pd	46	105	104.905079	$5/2^+$	22.2%
Cs	55	137	136.907073	$7/2^+$	30.2 yrs (β^-)
Ba	56	137	136.905812	$3/2^+$	11.2%
Tl	81	203	202.972320	$1/2^+$	29.5%
Os	76	191	190.960920	$9/2^-$	15.4 days (β^-)
Ir	77	191	190.960584	$3/2^+$	37.3%
Au	79	199	198.968254	$3/2^+$	16.8%

Question 1: Fundamental Forces 206

(a) Summarize the standard model of elementary particles. (9)

(b) If the proton-electron model of the nucleus were valid, show that all neutral atoms would contain an even number of fermions. (6)

(c) Give two examples of each:

i. Quarks (2)

ii. Leptons (2)

iii. Baryons (2)

iv. Mesons (2)

v. Unifying Theories (2)

Question 2: The Nuclear Atom 207

(a) Describe the main features of the Rutherford scattering formula. (6)

(b) Show, classically, that when an incident particle of mass m scatters of a target of mass m_T, the velocities are related by (Note: Lab frame) (7)

$$v_T^2\left(1-\frac{m_T}{m}\right) = 2\vec{v}_f\cdot\vec{v}_T$$

where $\vec{v}_T$ and $\vec{v}_f$ are the final velocities of the target and incident particles, respectively.

(c) Use the above result to do the following:

i. Show that the scattering angle is less than 90° for a beam of deuterons (non-relativistic) elastically scattering of a hydrogen target. (4)

ii. Show that there is no limit on the scattering angle for a beam of protons scattering of deuterons. (4)

iii. What are the limits on the scattering angle for a non-relativistic beam of deuterons scattering of a deuteron target? (4)

Question 3: Nuclear Properties 208

(a) Show that the dipole moment $\vec{p} = \int r\rho(r)dr$ is zero for a nucleus. (6)

(b) Show that the mean-square charge radius of a uniformly charged sphere is $\langle r^2 \rangle = 3R^2/5$ (5)

(c) One way of probing nuclear sizes is determining the size of charged matter by means of electron scattering experiments. The experimentally measurable quantity is the the electric form factor $F(\vec{q}) = \int \exp(i\vec{q}\cdot\vec{r})\rho(\vec{r})d^3r$, which is the Fourier transform of the charge distribution.

i. Show that if the charge distribution is spherically symmetric, the the form factor is given by (6)

$$F(q^2) = \frac{4\pi}{q}\int \sin(qr)\rho(r)r\,dr.$$

ii. Compute the form factor for $\rho(r) = \rho_0$ for $r < R$ zero otherwise. (8)

Question 4: Radioactivity 209

(a) An initial number $N_A(0)$ of nuclei A decay into daughter nuclei B, which is also radioactive. The respective decay probabilities are λ_A and λ_B. If $\lambda_B = 2\lambda_A$,

  i. Calculate the time (in terms of λ_A) when N_B is maximum. (6)

  ii. Calculate $N_B(max)$ in terms of $N_A(0)$. (4)

(b) Given that the production rate of a radioactive nuclide is P nuclei per second, derive the formula $N(t) = \frac{P}{\lambda}(1 - \exp(-\lambda t))$ for the production of the nuclide. (λ is the decay constant) (7)

(c) It is desired to determine the age of a wood timber used to construct an ancient shelter. A sample of the wood is analyzed for its ^{14}C content and gives 2.1 decays per minute. Another sample of the same size from a recently cut tree of the same type gives 5.3 decays per minute. What is the age of the sample? (Note: $t_{1/2} = 5730$ y) (8)

Question 5: β and γ Decay 2/0

(a) The maximum kinetic energy of the posistron spectrum emitted in the decay of a $^{11}C \rightarrow ^{11}B$ is 1.983 MeV. Use this information and the known mass of ^{11}B to calculate the mass of ^{11}C. (6)

(b) Supply the missing component(s) in the following processes:

i. $^{6}He \rightarrow ^{6}Li + e^{-} +$ (2)

ii. $^{40}K \rightarrow \nu +$ (2)

(c) Classify the following decays according to degree of forbiddenness:

i. $^{89}Sr(\frac{5}{2}^{+}) \rightarrow ^{89}Y(\frac{1}{2}^{-})$ (4)

ii. $^{36}Cl(2^{+}) \rightarrow ^{36}Ar(0^{+})$ (4)

(d) An even-Z, even-N nucleus has the following sequence of levels above its 0^{+} ground state: 2^{+} (89 keV), 4^{+} (288 keV), 6^{+} (585 keV), 0^{+} (1050 keV), 2^{+} (1129 keV). Draw an energy level diagram and show all reasonably probable γ transitions and their multipole assignments. (7)