UNIVERSITY OF SWAZILAND 211

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2013/2014

TITLE OF PAPER: STATISTICAL PHYSICS & THERMODYNAMICS

COURSE NUMBER: P461

TIME ALLOWED : THREE HOURS

ANSWER ANY FOUR QUESTIONS . ALL QUESTIONS CARRY EQUAL MARKS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

THIS PAPER CONTAINS FIVE QUESTIONS.

<u>**QUESTION ONE**</u>

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(a) (i) Explain briefly what is meant by **thermodynamic probability** of a system of particles. (3 marks)

(ii) What is the statistical significance of **thermodynamic probability** on the properties of the system? (2 marks)

(b) Four coins marked a, b, c, and d are tossed. If the number of heads (H) and the number of tails (T) obtained in a toss define a macrostate,

(i) Write down all the possible macrostates. (2 marks)

(ii) Calculate the number of microstates corresponding to each of the above macrostates. (4 marks)

$$W = \frac{N!}{\prod_s n_s!}$$

(iii) Assuming the number of coins is increased from 4 to 8, what would be the maximum number of microstates, that can be obtained in a toss. (2 marks)

(c) Explain briefly the difference between degenerate and non-degenerate energy levels. (2 marks)

(d) The allowed energies of a set of quantum particles is given as: $E = E_0 n^2$ where E_0 is a constant and $n^2 = n_x^2 + n_y^2 + n_z^2$, n_x, n_y, n_z being the quantum numbers representing an energy state. Given that the energy of a level is $26E_0$, find its degeneracy. (4 marks)

(e) (i) Define **density of states** of a system of particles. (2 marks)

(ii) Calculate the density of states in a sample of volume 10^{-4} m^3 at an energy level of 2.06 eV. (4 marks)

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$$

QUESTION TWO 2|3

(a) Derive the following Maxwell-Boltzmann distribution function for a system of classical particles in thermal equilibrium,

$$n_s = g_s \exp(\alpha + \beta\varepsilon_s),$$

where the symbols have their usual meanings.

(12 marks)

(b) A classical non-degenerate system has 15000 particles arranged in two energy levels having energies 1 unit and 2 units. The total energy of the system is 20,000 units. Find the values of α and β for the system and hence find the occupation number of the energy levels. Verify your results numerically.

(13 marks)

QUESTION THREE

(a) Show that the partition function of a classical perfect gas confined within a volume 'V' can be expressed as, $Z = \frac{V}{h^3}(2\pi mkT)^{3/2}$, where the symbols have their usual meanings.

[Given: density of states $g(\varepsilon)d\varepsilon = \frac{2\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon$; See Appendix for definite integrals]

(6 marks)

(b) Entropy of a classical perfect gas is expressed as $S = NkT\ln Z + \frac{E}{T}$, where 'Z' is the partition function and 'E' is the total energy = (3/2)NkT.

Two equal volumes of the same gas each having entropy 'S', and at the same temperature and pressure, are mixed together. Compute the entropy of the mixture in terms of 'S'. Do you see any anomaly in your result? If so, by deriving appropriate expressions, explain how the anomaly could be resolved.

(14 marks)

(c) Calculate the entropy of one mole of helium gas (^4He) at 300 K from the following data:

Molar volume of the gas	$= 22.4 \times 10^{-3}\ m^3mol^{-1}$
Avogadro number, N	$= 6.02 \times 10^{23}\ mol^{-1}$
Planck's constant, h	$= 6.63 \times 10^{-34}$ J.s
Mass of a He molecule	$= 6.65 \times 10^{-27}$ kg

(5 marks)

QUESTION FOUR

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(a) Max Planck derived the quantum statistical expression for the spectral distribution of energy from a black body:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT} - 1}$$

(i) Sketch a graph of E(λ) versus λ for any three temperatures $T_1 > T_2 > T_3$, according to this theory. State two important observations you make about these plots.

(ii) Show how this theory is superior to the classical Raleigh-Jeans theory and Wien's distribution law. (12 marks)

(b) (i) Use the above Planck's distribution function to show that the total energy radiated is proportional to the fourth power of the absolute temperature of the body. (8 marks)

(ii) Given that the proportionality constant in the above expression for the total energy is equal to σ (4/c), where 'σ' is the Stefan-Boltzmann constant, calculate the value of σ. (5 marks)

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QUESTION FIVE

(a) Given the density of states for a system of fermions,

$$g(\varepsilon)d\varepsilon = \frac{4\pi V}{h^3}(2m)^{3/2}\varepsilon^{1/2}d\varepsilon\ ,$$

show that the fermi energy is a function of the particle density of the system. (10 marks)

(b) (i) Explain why metals degenerate at room temperature. (4 marks)

(ii) At room temperature, the contribution of electrons in a metal towards its molar heat capacity is given as

$$C_v = 3Nk\frac{T}{T_F}$$

State and define T_F in the above equation. (2 marks)

"Metals have Fermi energy of the order of a few electron volts". Explain the significance of this statement as regards the contribution by electrons to the heat capacity of metals by giving an example. (5 marks)

(c) Calculate the Fermi energy of lithium, in electron volts. Lithium has an electron density of $5x10^{28}\ m^{-3}$. (4 marks)

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Appendix 1

Various definite integrals

$$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty e^{-ax^2}\,x\,dx = \frac{1}{2a}$$

$$\int_0^\infty e^{-ax^2}x^3\,dx = \frac{1}{2a^2}$$

$$\int_0^\infty e^{-ax^2}\,x^2\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}$$

$$\int_0^\infty e^{-ax^2}\,x^4\,dx = \frac{3}{8a^2}\left(\frac{\pi}{a}\right)^{1/2}$$

$$\int_0^\infty e^{-ax^2}\,x^5\,dx = \frac{1}{a^3}$$

$$\int_0^\infty \frac{x^3\,dx}{e^x - 1} = \frac{\pi^4}{15}$$

$$\int_0^\infty x^{1/2}\,e^{-\lambda x}\,dx = \frac{\pi^{1/2}}{2\lambda^{3/2}}$$

$$\int_0^\infty \frac{x^4 e^x}{(e^x-1)^2}\,dx = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^{1/2}}{e^x - 1}\,dx = \frac{2.61\pi^{1/2}}{2}$$

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Appendix 2

Physical Constants

Quantity	*symbol*	*value*
Speed of light	c	3.00×10^{8} ms^{-1}
Planck's constant	h	6.63×10^{-34} J.s
Stefan - Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Electronic charge	e	1.61×10^{-19} C
Mass of electron	m_e	9.11×10^{-31} kg
Mass of proton	m_p	1.67×10^{-27} kg
Gas constant	R	8.31 J mol^{-1} K^{-1}
Avogadro's number	N_A	6.02×10^{23}
Bohr magneton	μ_B	9.27×10^{-24} JT^{-1}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan constant	σ	5.67×10^{-8} $Wm^{-2}K^{-4}$
Atmospheric pressure		1.01×10^{5} Nm^{-2}
Mass of $_2^4$He atom		6.65×10^{-27} kg
Mass of $_2^3$He atom		5.11×10^{-27} kg
Volume of an ideal gas at STP		22.4 L mol^{-1}