UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND EGINEERING

DEPARTMENT OF PHYSICS

MAIN EXAMINATION 2015/2016

TITLE O F PAPER: MECHANICS

COURSE NUMBER: P211

TIME ALLOWED: THREE HOURS

INSTRUCTIONS: ANSWER ANY FOUR OUT OF FIVE QUESTIONS.

EACH QUESTION CARRIES 25 MARKS.

MARKS FOR EACH SECTION ARE IN THE RIGHT HAND MARGIN.

THIS PAPER HAS SIX PAGES INCLUDING THE COVER PAGE.

DO NOT OPEN THE PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE CHIEF INVIGILATOR.

QUESTION 1

(a) Derive the basic kinematic equation:
$x_0{}^2 = v_0{}^2 + 2a(x - x_0)$. **(5 marks)**

(b) The direction cosines of a vector are the cosines of the angles it makes with the coordinate axis. The cosines of the angles between a vector $\vec{A}$ and the x, y, and z axes in Cartesian coordinates are usually called in turn, α, β, and γ. Using vector algebra, prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$. **(6 marks)**

(c) With the aid of a diagram make use of vector algebra to prove the identity
$\cos(A - B) = \cos A \cos B + \sin A \sin B$. **(6 marks)**

(d) A tyre of radius R roles in a straight line without slipping. Its centre moves at constant velocity V. A small stone lodged at the circumference of the tyre touches the road at time $t = 0$. The position of the stone at some later time is illustrated in Figure 1. Find the

i. position, **(4 marks)**
ii. velocity, and **(2 marks)**
iii. acceleration of the stone as a function of time. **(2 marks)**

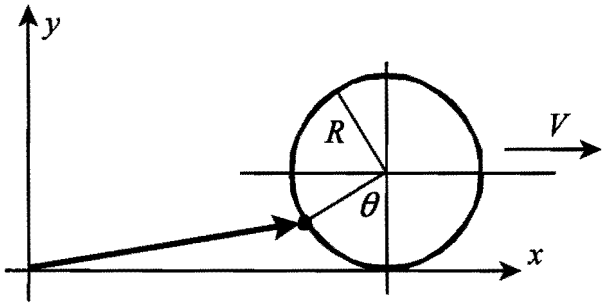

Figure 1.

QUESTION 2

(a) The pedagogical machine illustrated in Figure 2 has a force F applied to the mass m_1. All surfaces are frictionless. Find the acceleration of the mass m_1 if the force F is equal to zero. **(8 marks)**

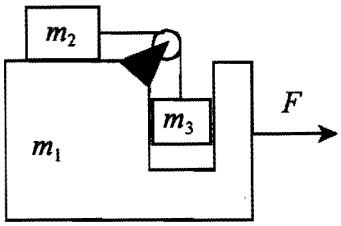

Figure 2.

(b) A block of mass m rests on a wedge inclined at an angle θ. The coefficient of static friction between the block and the wedge is μ. The wedge is give an acceleration a as shown in Figure 3. The angle θ is such that the mass m would slide down the wedge if the acceleration a was zero. Gravitational acceleration g is downward as shown. Find the minimum acceleration for the block to remain on the wedge without sliding. **(7 marks)**

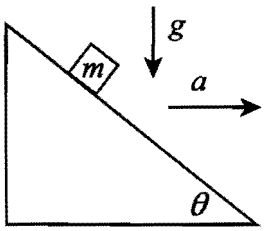

Figure 3.

(c) A mass m whirls around on a string which passes through a ring as shown in Figure 4. Initially, the mass is a distance r_0 from the centre and is revolving at angular velocity ω_0. The string is pulled starting at $t = 0$, so that the radial distance of the mass from the centre decreases. Find the angular velocity of the mass at later times. **(10 marks)**

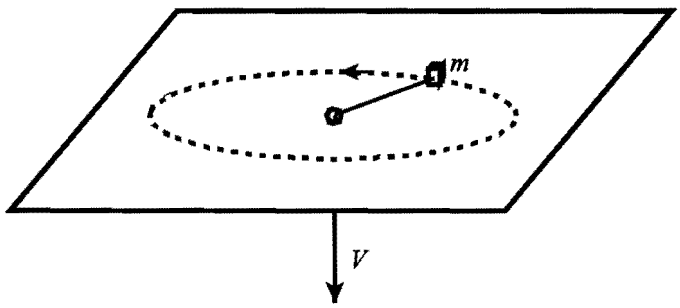

Figure 4.

QUESTION 3

(a) Starting with an element of area find the centre of mass of a semicircular flat plate of radius R with its centre at the origin as shown in Figure 5. **(9 marks)**

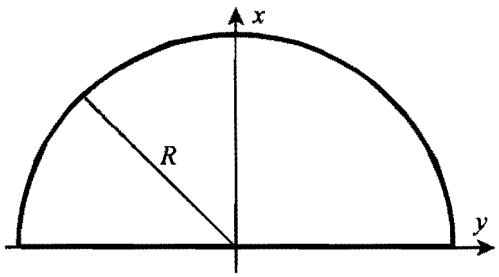

Figure 5.

(b) A continuous stream of particles of cross sectional area A and density ρ is incident to a stationary flat surface with a velocity of v. Find the force on the surface in terms of A, ρ and v if the stream rebounds with a speed equal to the incident speed. **(6 marks)**

(c) A rocket ascends from rest in a uniform gravitational field of magnitude g by ejecting exhaust gases with a constant speed of u. The mass is expelled at the rate $\frac{dM}{dt} = \gamma M$, where M is the instantaneous mass of the rocket and γ a constant. The rocket is retarded by air resistance force equal to Mbv, where v is the instantaneous velocity of the rocket and b is a constant. Find

i. the time differential equation for the velocity of the rocket, **(8 marks)**

ii. the terminal velocity of the rocket if it reaches it before the depletion of the fuel. **(2 marks)**

QUESTION 4

(a) A small block slides from rest from the top of a frictionless sphere of radius R as shown in Figure 6. How far below the top of the sphere x does it lose contact with the sphere? **(7 marks)**

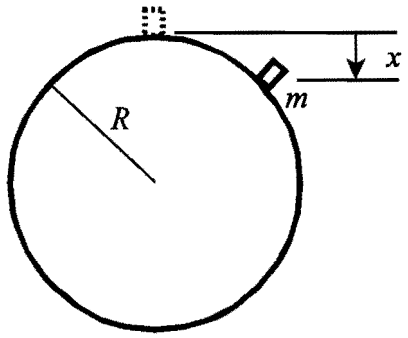

Figure 6.

(b) The force $\vec{F} = A(x^2\hat{\imath} + xy\hat{\jmath})$, where A is a constant with appropriate unit, is used to move a particle around the unit circle from (0,0) to (2,0). Find the work done in moving the particle along the semicircle path a and also along the straight line path b. See Figure 7. Comment on the results obtained. **(12 marks)**

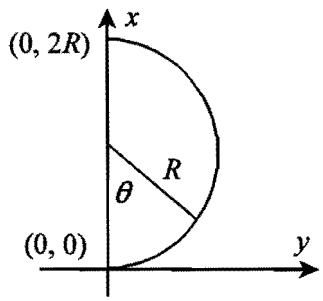

Figure 7.

(c) Derive an expression for the relationship between potential energy and force. **(6 marks)**

QUESTION 5

(a) Find the moment of inertia of a thin uniform right angled triangular plate about the y-axis. The plate is arranged as shown in Figure 8 with base b and height h. **(8 marks)**

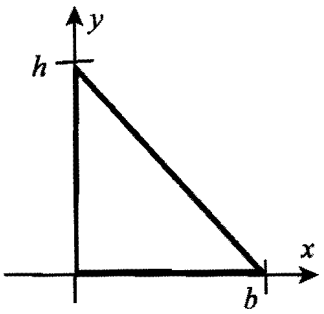

Figure 8.

(b) A spaceship is sent to investigate a planet of mass m and radius R. At a distance $5R$ from the centre of the planet it stops relative to the planet, and then fires an instrument probe with speed v_0. See Figure 9. The mass m of the probe is much smaller than the mass of the planet. Find the maximum angle at which the probe can be launched to just graze the planet, assuming that the gravitational field of the planet is constant within the distance travelled, in terms of M, R, and v_0. **(9 marks)**

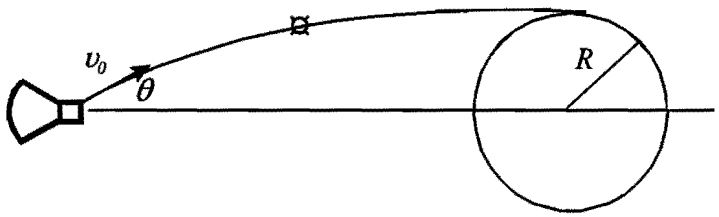

Figure 9.

(c) A uniform drum of radius b and mass M rolls without slipping down a plane inclined at an angle θ. See Figure 10. Find its acceleration down the plane in terms of the acceleration due to gravity g and the angle θ, using torque and angular momentum. **(8 marks)**

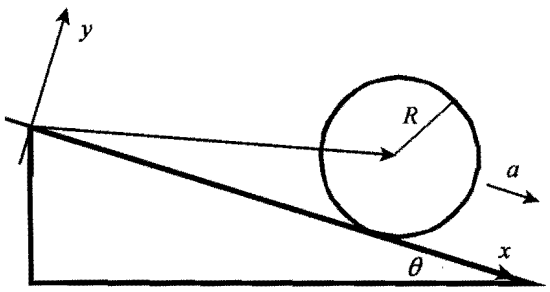

Figure 10.