UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF PHYSICS

SUPPLEMENTARY EXAMINATION: 2015/2016

TITLE OF PAPER: ELECTRICITY AND MAGNETISM

COURSE NUMBER: P221

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

- ANSWER ANY FOUR OUT OF THE FIVE QUESTIONS.
- EACH QUESTION CARRIES 25 POINTS.
- POINTS FOR DIFFERENT SECTIONS ARE SHOWN IN THE RIGHT-HAND MARGIN.
- USE THE INFORMATION IN THE NEXT PAGE WHEN NECESSARY.

THIS PAPER HAS 7 PAGES, INCLUDING THIS PAGE.

DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Useful Mathematical Relations

Gradient Theorem

$$\int_{\vec{a}}^{\vec{b}} (\nabla f) \cdot d\vec{l} = f(\vec{b}) - f(\vec{a})$$

Divergence Theorem

$$\int \nabla \cdot \vec{A} d\tau = \oint \vec{A} \cdot d\vec{a}$$

Curl Theorem

$$\int (\nabla \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Line and Volume Elements

Cartesian: $d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$, $d\tau = dxdydz$

Cylindrical: $d\vec{l} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}$, $d\tau = sdsd\phi dz$

Spherical: $d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$, $d\tau = r^2 \sin\theta dr d\theta d\phi$

Gradient and Divergence in Spherical Coordinates

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Dirac Delta Function

$$\nabla \cdot \left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

Question 1: Electrostatics ....

(a) Use the electric field of a point charge to show directly that the electrostatic field is irrotational.

(b) A sphere of radius b with a concentric cavity of radius a has a volume charge density that varies with distance r from the center according to $\rho(r) = \rho_0 r^{1/2}$ where ρ_0 is a constant.

i. How much charge is enclosed in a sphere of radius r such that $r < a$?

ii. What is the electric field inside the cavity?

iii. For a sphere of radius r such that $a < r < b$ calculate the amount of charge enclosed.

iv. What is the electric field at a point within the spherical shell, i.e the region where $a < r < b$?

v. How much charge is enclosed in a sphere of radius r such that $r > b$?

vi. What is the electric field at a point outside the sphere.

Question 2: Electrostatic II ....

Charge is distributed over the surface of a circular disk of radius a lying in the xy plane with the origin at the center. The surface density is given in cylindrical coordinates by $\sigma = As^2$, where A is a constant.

(a) What are the units of A?

(b) What is the expression for the total charge on the disk?

(c) Find the force produced by this charge distribution on a point charge located on the z axis.

(d) What is the force on the disk due to the point charge?

(e) Use the field and a suitable reference point to find the electrostatic potential at a point on the z axis.

Note

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = \ln\left[2\left(x+\sqrt{a^2+x^2}\right)\right]$$

$$\int \frac{x}{\sqrt{a^2+x^2}}dx = \sqrt{a^2+x^2}$$

$$\int \frac{2x^2+a^2}{\sqrt{a^2+x^2}}dx = x\sqrt{a^2+x^2}$$

$$\int \frac{x^3}{\sqrt{a^2+x^2}}dx = \frac{1}{3}(x^2-2a^2)\sqrt{a^2+x^2}$$

$$\int \frac{x^3}{(a^2+x^2)^{3/2}}dx = \frac{2a^2+x^2}{\sqrt{a^2+x^2}}$$

Question 3: Magnetostatics . . . .

(a) Describe the mechanism responsible for paramegnetism.

(b) Describe the mechanism responsible for diamagnetism.

(c) Deduce that the normal component of a magnetostatic field is always continuous across a boundary surface.

(d) Deduce that the tangential component of a magnetic field is discontinuous across a current carrying boundary surface.

(e) What is the general expression for the discontinuity of the magnetic field across a current carrying boundary surface?

(f) What is the boundary condition on the vector potential $\mathbf{A}$?

Question 4: Magnetostatcs II ....

(a) Derive the continuity equation

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t}$$

(b) Suppose a current I is uniformly distributed over a wire of circular cross section with radius a.

i. Determine the volume current density $\mathbf{J}$.

ii. What is the corresponding charge density?

(c) Use Ampere's law to find the magnetic field a distance s from a long straight wire, carrying a steady current I.

(d) Use Ampere's law to find the magnetic field of a very long solenoid, consisting of n closely wound turns per unit length on a cylinder of radius a.

Question 5: Circuits and Electrodynamics ....

Consider a square loop of wire (side a) lying on a table, a distance s from a very long straight wire, which carries a current I.

(a) Use the Biot-Savart law to verify that the field $\mathbf{B}$ a distance s from the wire is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

(b) Find the flux of $\mathbf{B}$ through the square loop.

(c) If someone pulls the loop directly away from the wire, at speed v, what emf is generated?

(d) Draw a diagram indicating in which direction an induced current flows as the wire is pulled away.

(e) What emf is generated when the loop is pulled, at speed v, parallel to the wire?